

Air loading effect on the resonance frequencies of a piano soundboard modeled by a rectangular orthotropic plate

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1. Introduction

Various studies of piano soundboard vibrations and sound radiations have been reported [1–3]. Most of them are theoretical and experimental studies on modes and frequencies of soundboard vibrations. Relatively fewer experimental studies have been conducted on sound radiations from soundboards. Measurements of radiated power and radiation efficiencies of a piano soundboard had been discussed in Ref. [4].

One missing but very important topic is the effect of air loading on the resonance frequencies of piano soundboards. Finite element methods (FEM) are commonly used for theoretical modal analysis without taking into account the effect of air loading, and the resonance frequencies are compared to experimental results of real piano soundboards with air loading. This is permissible only if the effect of the air loading is negligible. For example, in Ref. [5], theoretical resonance frequencies higher than corresponding experimental results had been obtained. There is a possibility that this difference can be removed or lessened if the air loading is taken care of. Recently, Takasawa proposed physical models of piano soundboards using a finite difference method. However, his models do not include the effect of air loading [6,7].

The aim of this study is to find out the degree of the effect of air loading on the resonance frequencies of a piano soundboard. In order to serve this purpose for the time being, we may model the soundboard as a simply-supported rectangular orthotropic plate with an infinite baffle. Although the condition of the infinite baffle is different from that of a real piano soundboard, the use of an infinite baffle may be appropriate above 70 Hz or at least above 100 Hz, as it was mentioned in Ref. [2] that the cancellation (dipole effect) between the sound pressures radiated from the top and bottom sides of the soundboard usually does not occur above 70 Hz. Note also that the air mass-loading at low frequencies that may reduce resonance frequencies can be severer in a real soundboard where the infinite baffle is absent. This is based on the theoretical results that the reactance at one side of a vibrating piston mounted in an infinite baffle is significantly less than that on the exterior side of the same piston mounted at one end of a long pipe (without a baffle) at low frequencies, or when $ka < 1$ (where a is the radius of the piston) [8].

2. Specifications of the rectangular orthotropic plate

Parameters of the rectangular orthotropic plate and air used for the calculation are shown below. The plate has the area of 1 m^2 . A relatively large thickness (0.009 m) is adopted considering the effect of ribs. The values shown in the parenthesis will be always used in the calculation. Material properties not shown will be given later.

L_x : length in the x -direction (1.2 m)

L_y : length in the y -direction (0.83 m)

h : thickness of the plate (0.009 m)

ρ : density of the plate (400 kg/m³)

E_x : Young's modulus in the x -direction

E_y : Young's modulus in the y -direction

ν_{xy}, ν_{yx} : major and minor Poisson's ratios

G_{xy} : Shear modulus associated with the x - y plane

LF : loss factor (0.025)

ρ_a : density of air (1.21 kg/m³)

c : sound speed of air (343 m/s)

3. Theoretical discussion

3.1. Vibration of the plate

The equation of transverse motion of an orthotropic plate under a time-harmonic pressure distribution $Q(x, y)$ at angular frequency ω is given by [9]

$$D_x \frac{\partial^4 W}{\partial x^4} + (\nu_{yx} D_x + \nu_{xy} D_y + 4D_{xy}) \frac{\partial^4 W}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 W}{\partial y^4} - \rho h \omega^2 W = Q(x, y) \quad (1)$$

where $W(x, y)$ is the transverse displacement of the plate.

$$D_x = E_x h^3 / 12(1 - \nu_{yx} \nu_{yx}), \quad D_y = E_y h^3 / 12(1 - \nu_{xy} \nu_{yx}),$$

$$D_{xy} = h^3 G_{xy} / 12, \quad (2)$$

are the bending stiffness in the respective directions and the relation

$$E_x / E_y = \nu_{yx} / \nu_{xy} \quad (3)$$

holds for an orthotropic plate. In Eq. (1), if we define an operator L as

$$L = D_x \frac{\partial^4}{\partial x^4} + (\nu_{yx} D_x + \nu_{xy} D_y + 4D_{xy}) \frac{\partial^4}{\partial x^2 \partial y^2} + D_y \frac{\partial^4}{\partial y^4} - \rho h \omega^2 \quad (4)$$

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applied to the displacement function $W(x, y)$, the displacement response to the external excitation is

$$W = L^{-1}[Q(x, y)] \quad (5)$$

It is known that a combination of eigen (modal) functions shown below satisfies a simply-supported boundary condition and gives a solution of Eq. (1)

$$W(x, y) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} A_{mn} \sin\left(\frac{m\pi x}{L_x}\right) \sin\left(\frac{n\pi y}{L_y}\right),$$

$m, n: \text{integers } (\geq 1)$ (6)

where $\{A\}$ is a set of coefficients for the eigen functions. Substituting Eq. (6) into Eq. (1), Eq. (1) becomes

$$\sum_{n=1}^{\infty} \sum_{m=1}^{\infty} (D_{mn} - \rho h \omega^2) A_{mn} \sin\left(\frac{m\pi x}{L_x}\right) \sin\left(\frac{n\pi y}{L_y}\right) = Q(x, y) \quad (7)$$

where

$$D_{mn} = D_x \left(\frac{m\pi}{L_x}\right)^4 + 2(\nu_{xy} D_y + 2D_{xy}) \left(\frac{m\pi}{L_x}\right)^2 \left(\frac{n\pi}{L_y}\right)^2 + D_y \left(\frac{n\pi}{L_y}\right)^4 \quad (8)$$

By the use of the orthogonality of the eigen functions, A_{mn} is given by

$$A_{mn} = \int_{y=0}^{L_y} \int_{x=0}^{L_x} Q(x, y) \sin\left(\frac{m\pi x}{L_x}\right) \sin\left(\frac{n\pi y}{L_y}\right) dx dy \Bigg/ \left[(D_{mn} - \rho h \omega^2) \frac{L_x L_y}{4} \right] \quad (9)$$

where

$$\int_{y=0}^{L_y} \int_{x=0}^{L_x} \sin^2\left(\frac{m\pi x}{L_x}\right) \sin^2\left(\frac{n\pi y}{L_y}\right) dx dy = L_x L_y / 4 \quad (10)$$

was used. If a point force $F_0 \delta(x - x_0) \delta(y - y_0)$ is applied at (x_0, y_0) , A_{mn} is given by

$$A_{mn} = F_0 \sin\left(\frac{m\pi x_0}{L_x}\right) \sin\left(\frac{n\pi y_0}{L_y}\right) \Bigg/ \left[(D_{mn} - \rho h \omega^2) \frac{L_x L_y}{4} \right] \quad (11)$$

Resonance frequencies are given from Eq. (11) by

$$\omega_{mn} = \sqrt{D_{mn} / \rho h} \quad (12)$$

The ratio of the velocity response at (x, y) to the force $F_0 \delta(x - x_0) \delta(y - y_0)$ gives a transfer admittance

$$j\omega W(x, y) / F_0 = j\omega \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \sin\left(\frac{m\pi x_0}{L_x}\right) \sin\left(\frac{n\pi y_0}{L_y}\right) \times \sin\left(\frac{m\pi x}{L_x}\right) \sin\left(\frac{n\pi y}{L_y}\right) \Bigg/ \left[(D_{mn} - \rho h \omega^2) \frac{L_x L_y}{4} \right] \quad (13)$$

If $(x, y) = (x_0, y_0)$, Eq. (13) gives the driving-point admittance.

For simplicity of notation, eigen functions are represented by U_i , where the combination of m and n is represented by a

single integer i (mode number), and a position on the plate (x, y) is represented by s . The numbering is taken in the ascending order of the resonance frequencies. Equation (6) is then represented by

$$W(s) = \sum_{i=1}^{\infty} A_i U_i(s), \quad i: \text{integer } (\geq 1) \quad (14)$$

Also, Eq. (9) becomes

$$A_i = \int_S \left[Q(s) U_i(s) \Bigg/ \left\{ (D_i - \rho h \omega^2) \frac{L_x L_y}{4} \right\} \right] ds \quad (15)$$

The damping of the plate is taken into account by replacing D_i by $D_i(1 + jLF)$ in the calculation, where LF is the loss factor.

3.2. Sound radiation from the plate

If the plate in an infinite baffle is in the air and driven by a point force $F_0 \delta(s - s_0)$, the equation of motion is given by

$$L[W] = F_0 \delta(s - s_0) - 2P(s) \quad (16)$$

The sound pressure $P(s)$ on one side of the plate is given by the velocity distribution $j\omega W(s)$ as

$$P(s) = \frac{j\omega \rho_a}{2\pi} \int_S j\omega W(s') \frac{\exp(-jkd)}{d} ds' = \beta[W] \quad (17)$$

where d is the distance from the surface element ds' to the observation point s , and β is an operator that gives a pressure when applied to a displacement function. By combining Eqs. (16) and (17), the following equation is obtained [10].

$$W + 2L^{-1} \beta[W] = L^{-1}[F_0 \delta(s - s_0)] \quad (18)$$

By expressing W by Eq. (14) with unknown coefficient vector $\{A\}$, the second term of the left-hand side of Eq. (18) is given by

$$2L^{-1} \beta[W] = \int_S 2P(s) U_i(s) / [(D_i - \rho h \omega^2)(L_x L_y / 4)] ds = \sum_{i=1}^{\infty} A_i (-\omega^2 \rho_a / \pi) \int_S \int_{S'} [U_j(s') U_i(s)] \times \{ \exp(-jkd) / d \} / \{ (D_i - \rho h \omega^2)(L_x L_y / 4) \} ds' ds = \sum_{i=1}^{\infty} c_{ji} A_i \quad (19)$$

The right-hand side of Eq. (18) is given from Eq. (11) by

$$B_i = F_0 U_i(s_0) \Bigg/ \left[(D_i - \rho h \omega^2) \frac{L_x L_y}{4} \right] \quad (20)$$

By the use of Eqs. (18), (19) and (20), and by truncating the order of eigen functions by I , a set of $I \times I$ simultaneous equations with unknown coefficients $\{A\}$ is obtained.

$$\begin{bmatrix} 1 + c_{11} & c_{12} & \cdots & c_{1I} \\ c_{21} & 1 + c_{22} & \cdots & c_{2I} \\ \cdots & \cdots & \cdots & \cdots \\ c_{I1} & c_{I2} & \cdots & 1 + c_{II} \end{bmatrix} \begin{Bmatrix} A_1 \\ A_2 \\ \vdots \\ A_I \end{Bmatrix} = \begin{Bmatrix} B_1 \\ B_2 \\ \vdots \\ B_I \end{Bmatrix} \quad (21)$$

where c_{ji} represents an effect of pressure excited by the j -th mode to the displacement of the i -th mode. The unknown coefficient vector $\{A\}$ is obtained by solving Eq. (21).

Table 1 Parameters used for the three cases.

	Case I	Case II	Case III
E_x	10 GPa	20 GPa	10 GPa
E_y	10 GPa	10 GPa	20 GPa
G_{xy}	3.7 GPa	7.4 GPa	7.4 GPa
ν_{xy}	0.35	0.35	0.175
ν_{yx}	0.35	0.175	0.35

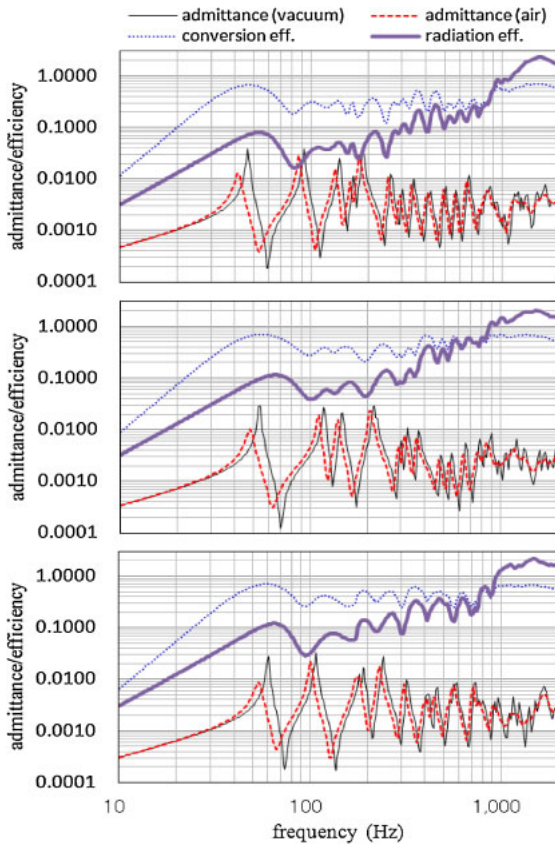


Fig. 1 Driving-point admittances (m/s/N) in vacuum and in air and conversion and radiation efficiencies for Cases I (top), II (middle), and III (bottom).

4. Numerical results and discussion

Driving point admittances at $(0.252L_x, 0.252L_y)$ in vacuum and in air were calculated for three cases of parameters shown in Table 1. Values of other parameters are already given in Sect. 2. The Young’s modulus of 10 GPa is given for the Young’s modulus of spruce in the grain direction. The Young’s modulus of 20 GPa is given assuming the case of over-stiffening. $G_{xy} = E_x/2(1 + \nu_{xy})$ ($E_x > E_y$) or $G_{xy} = E_y/2(1 + \nu_{yx})$ ($E_y > E_x$) was used to determine G_{xy} . Note that Case I is for an isotropic plate.

Figure 1 shows the driving-point admittances in vacuum and in the air for Cases I, II, and III. Conversion and radiation efficiency characteristics are also shown in Fig. 1. The real part of the driving-point admittance (m/s/N) is equal to the input power (N·m/s) if $F_0 = 1$ (N). Since the velocity and the pressure distributions on the surface are calculated, the radiated power is also obtained. The conversion efficiency

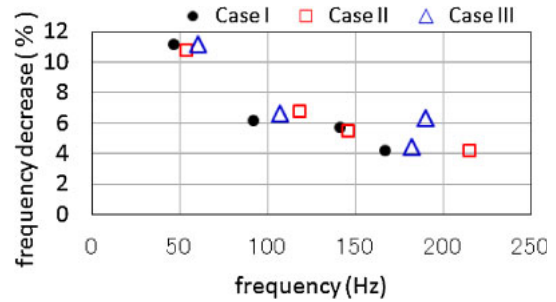


Fig. 2 Resonance frequency decreases of the first four modes for Cases I, II, and III.

is defined as the ratio of the radiated power (input power minus the power loss to damping) to the input power. The radiation efficiency (normalized radiation resistance) is defined as the ratio of the radiated power to the power radiated with an assumption that the plate has the radiation resistance of ρc on the whole surface.

Figure 1 clearly shows that the effect of air loading is non-negligible. The calculated conversion efficiency is roughly equal to the measured efficiency in Ref. [4]. (Note that, in Ref. [4], the radiated power from one side of the soundboard was taken into account.) The result that the radiation efficiency approaches 1 or becomes larger between 1 and 2 kHz was also found in Ref. [4]. As the frequency increases, the driving-point admittances in Fig. 1 approach the inverse of the characteristic impedance given in [11].

Decreases of resonance frequencies from those in vacuum to those in the air of the first four modes for the three cases are shown in Fig. 2. The horizontal axis indicates the resonance frequencies in vacuum. The results show that the frequency decreases for the first four modes due to air loading are on the order of several percent or more if the frequencies are below 200 Hz. Note that m and n of the four modes for Case I, II, and III are [(1, 1), (2, 1), (1, 2), (3, 1)], [(1, 1), (2, 1), (1, 2), (2, 2)], and [(1, 1), (2, 1), (3, 1), (1, 2)] in the ascending order of resonance frequencies, respectively.

5. Conclusions

The numerical evaluation of the effect of air loading on the resonance frequencies of an orthotropic rectangular plate in an infinite baffle shows that the air loading effect of piano soundboard should not be neglected. The order of the calculated resonance frequency reduction can be as much as 10% or more below 100Hz. Those found in Ref. [5] are severer. For an accurate evaluation of FEM modeling of piano soundboard vibration and sound radiation, the interaction between the soundboard and the air should be taken into account.

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