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An Accurate Measurement of Rotation Velocity-Eliminating Measurement Errors Caused by Gear Tooth Shape Unevenness

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ABSTRACT

It is very important to accurately measure rotation frequencies and fluctuations of rotating systems since they cause vibrations and noises, and since they sometimes indicate system malfunctions. Most rotating systems are equipped with electro- or magneto-conductive gears as their components, and rotation pulses are very commonly obtained by installing electromagnetic or electrostatic type sensors closely to target gears, and time dependent (instantaneous) rotation frequencies are obtained from intervals between adjacent pulses. However, since the number of pulses per revolution is relatively small, a method of obtaining instantaneous frequencies from adjacent pulse intervals is not adequate. For these kinds of pulses, instantaneous rotation frequencies are typically derived using the analytic signal (or Hilbert transform) method. In either case, there is an inherent limitation in using rotation pulses obtained from gears. Individual shapes of teeth may slightly differ from each other and this unevenness of the tooth shape causes errors in the instantaneous rotation frequency estimation.

In order to solve the above mentioned problem, this paper proposes a method that uses two sensors closely located to each other. Information regarding the true rotation frequencies of the gear is contained in the two sensor signals without delay. On the other hand, the errors (spurious instantaneous frequencies) caused by the unevenness of the tooth shapes are contained with a time delay corresponding to the angle difference between the sensors. The subtraction of the instantaneous frequencies obtained from the two rotation pulses gives an instantaneous rotation frequency signal that contains only the spurious instantaneous frequencies. Since the time delay can be known from the phase measurement of the two pulses, the spurious instantaneous frequencies (and then the true instantaneous frequencies) can be calculated. This paper includes numerical discussions of the effectiveness of the method and some experimental results.

INTRODUCTION

It is very important to accurately measure rotation frequencies (velocities) and fluctuations of a rotating system since they are critical for identifying and debugging vibrations and noises, and since they sometimes indicate system malfunctions. One of methods to measure the rotation frequencies including higher order fluctuations with high accuracy is to use rotary encoders which generate as many as several tens of thousands of pulses per revolution. Time dependent (instantaneous) rotation frequencies are obtained from intervals between adjacent pulses. However, measurement systems tend to be expensive and the installation of encoders is sometimes impossible for in-situ measurements.

Since most rotating systems are equipped with electro- or magneto-conductive gears as their components, rotation pulses are very commonly obtained by installing electromagnetic, electrostatic, or eddy-current type sensors close to target gears. Those gears may have several tens to a couple of hundreds of teeth. Since the number of pulses per revolution is relatively small, the direct method to obtain instantaneous frequencies from adjacent pulse intervals is not adequate. For these kinds of rotation pulses, instantaneous rotation frequencies are typically derived using the analytic signal (or Hilbert transform) method [$\underline{1}$, $\underline{2}$, $\underline{3}$, $\underline{4}$, $\underline{5}$]. The original rotation pulse signals obtained by those sensing systems are roughly sinusoidal but sometimes they are converted to rectangular pulse signals for ease of later use. However, reference [$\underline{5}$] showed that the former is more suitable than the latter to obtain instantaneous rotation frequencies by the analytic signal method. Applications of the analytic signal method to various signals were discussed in reference [$\underline{6}$].

There is an inherent limitation in using gears to generate rotation pulses. Individual shapes of teeth may slightly differ from each other and this unevenness of the tooth shape causes errors in the instantaneous rotation frequency analysis. The authors are unaware of any research published addressing the removal of this type of error. The present paper proposes a method of removing or at least reducing rotation frequency estimation errors caused by unevenness of tooth shapes of a gear using two sensors closely located to each other.

Needless to say, the proposed method is applicable to rotation pulse detecting systems with optoelectric sensors or other types of sensing systems. It is also noted that the proposed method is independent of the measurement method to obtain instantaneous frequencies from two rotation pulse signals.

PROPOSED METHOD

In this section, a method of removing instantaneous rotation frequency estimation errors caused by the unevenness of tooth shapes of a gear is described. The proposed method uses two sensors as shown in Fig. 1.



Figure 1. Proposed rotation pulse detecting system.

Sensors 1 and 2 generate rotation pulse signals $x_1(n)$ and $x_2(n)$, respectively, from which two instantaneous rotation frequencies $f_1(n)$ and $f_2(n)$ are obtained.

A method of obtaining $f_1(n)$ and $f_2(n)$ from $x_1(n)$ and $x_2(n)$ needs not be specified to apply the proposed method. The functions $f_1(n)$ and $f_2(n)$ consist of "true" instantaneous frequency $f_{tr}(n)$ and "spurious" instantaneous frequencies f_{sp} ,

 $_1(n)$ and $f_{sp,2}(n)$, respectively. The true instantaneous frequency $f_{tr}(n)$ should appear at the same time in $f_1(n)$ and $f_2(n)$. The spurious instantaneous frequencies represent the errors caused by the unevenness of the tooth shapes of the gear. It is assumed that $f_{sp,2}(n)$ is delayed by M from $f_{sp,1}(n)$. Note that M is not necessarily an integer. Then, $f_1(n)$ and $f_2(n)$ are given, respectively, by

$$f_{\rm l}(n) = f_{\rm tr}(n) + f_{\rm sp}(n)$$
 (1)

$$f_2(n) = f_{\rm tr}(n) + f_{\rm sp}(n-M) \tag{2}$$

The subtraction of Eq. (2) from Eq. (1) gives

$$f_{1-2}(n) = f_{sp}(n) - f_{sp}(n-M)$$
(3)

Equation (3) is discrete Fourier transformed:

$$F_{1-2}(k) = F_{\rm sp}(k) [1 - e^{-j2\pi kM/N}]$$
(4)

In the derivation of Eq. (4), the property that the *N*-point discrete Fourier transform of f(n-m) is given by $F(k)e^{-j2\pi km/N}$ is used. The spurious instantaneous frequency is then given by

$$f_{sp}(n) = \text{IDFT}[F_{12}(k) / \{1 - e^{-j2\pi kMN}\}]$$
(5)

where IDFT indicates the inverse discrete Fourier transform. Then the true instantaneous frequency is given by Eq. (1) and Eq. (5) as

$$f_{tr}(n) = f_1(n) - f_{sp}(n)$$
 (6)

The above method is very simple and straightforward, but one difficulty of this method is foreseen. There is a division by $\{1-e^{-j2\pi kM/N}\}$ in Eq. (5), which may become or approach 0 at some frequencies.

ROTATION PULSE GENERATION MODEL

In the papers $[\underline{4},\underline{5}]$, rotation pulse generation models are used that can produce rotation pulse signals with known instantaneous frequencies and phases. These kinds of pulse

signal generation models are a must for the accurate evaluation of rotation frequency analysis methods.

A tooth shape of a gear used for the rotation pulse generation model in the present study is shown in Fig. 2 (only one sensor is shown). The gear of the model has 120 teeth (each tooth width is 3 degrees). An electrostatic sensor has a linear sensing area (length) of 1 degree arc (with uniform sensitivity)-i.e., the average distance from the sensor to the gear teeth within ± 0.5 degrees from the center of the sensing area is output from the sensor. To simplify the calculation of the distance from the sensor to the gear at an arbitrary angle, it is assumed that the sensing arc of the sensor and the circumference of the gear are parallel as shown in Fig. 2. Note that absolute values of r_1 and r_2 are irrelevant since the DC component is removed from the sensor output and gains of the sensors have no effect on the instantaneous frequencies obtained from the pulse signals.



Figure 2. Tooth shape from 0 to 3 degrees of a model gear with 120 teeth.

The instantaneous rotation frequency f(n) of the gear with a rotation fluctuation as a function of time $n\Delta t$ can be expressed by

$$f(n) = f_0 + \Delta f \sin(2\pi f_m n \Delta t)$$
⁽⁷⁾

and the phase is given by:

$$\phi(n) = 2\pi\Delta t \sum_{m=0}^{n} f(m) + \phi_0$$
(8)

where ϕ_0 is an initial phase, which can be assumed to be zero. If the total angle (phase) of the gear rotation is known, the average distance from the sensor to the gear is obtained. In this study, the average distance is obtained by calculating distances at 51 equally spaced points along the sensing area.

An example of waveforms generated by the pulse generation model for the case with $f_0 = 30$, $\Delta f = 0$, M = 20, $f_s = 51,200$ is shown in Fig. 3. Since $M_Z = 14.222 (=51,200/(30 \times 120))^{\dagger}$, the

sensors are separated by approximately 4.22 degrees. As <u>Fig.</u> <u>3</u> shows, the sensor 2 output waveform is almost identical with the sensor 1 output.



Figure 3. Example of rotation pulse signals obtained by the two sensors for the case with $f_0 = 30$, $\Delta f = 0$, M = 20, and $f_s = 51,200$.

If the gear is made perfectly as shown in Fig. 2, it will be referred to as the "ideal gear". In the present study, the "faulty gear" is defined as follows: Each tooth shape from 0 to 3 degrees shown in Fig. 2 is widened by 0.01 %, and accordingly, the flat portion of the last tooth (originally 1.0 degree width) is shortened to 0.964 degrees. Since the pulse width of the 120^{th} tooth is narrower than other pulses, the instantaneous frequency should increase when this tooth comes under the sensing area. This increase (or decrease) is the rotation frequency estimation error caused by the unevenness of the tooth shapes of the gear.

ANALYTIC SIGNAL METHOD

In the present study, the instantaneous rotation frequency is obtained from rotation pulse signals by the analytic signal method [1, 2, 3, 4, 5, 6]. In this section, the procedure is briefly given using the rotation pulse signals generated by the model described in the previous section. The rotation pulse signals are discrete Fourier transformed and all spectra except for positive frequency components including the tooth frequency component f_Z are removed. In the present study, the low and the high cutoff frequencies of the rectangular filter are set equal to $(1-c_b) f_Z$ and $(1+c_b) f_Z$, respectively, with $c_b = 0.7$. Fig. 4 shows the spectrum of the rotation pulse signal of sensor 1 for the case with $f_{\rm m} = 300$ and $\Delta f = 1$. The spectral peak is at the tooth frequency f_Z . Note that the carrier frequency of the rotation pulse signal is f_Z instead of f_0 (see [5]). The reason why the high cutoff frequency is set closer to the second harmonic $(2 f_Z)$ instead of the center between f_Z and 2 f_Z is that the level of the 2 f_Z component and its sidelobes are lower than those of the fundamental (f_Z) component. The frequency band from 1080 Hz to 6120 Hz (range within the two vertical lines in Fig. 4) contains the

[†]Please refer to the DEFINITIONS/ABBREVIATIONS section at the end of this document for a definition of MZ and other terms used throughout this paper.

carrier component at f_Z (= 3,600) and side lobes at $f_Z \pm j f_m$ (*j* : integer). The spectrum is inverse Fourier transformed back to the time domain to obtain an analytic signal.



Figure 4. Power spectrum of the rotation pulse signal of sensor 1 for the case with $f_m = 300$ and $\Delta f = 1$.

Figure 5 shows a portion of the analytic time signal on the complex plane. The instantaneous frequency at discrete time $n\Delta t$ is obtained by dividing the phase increment from (n-1) to (n+1) by $4\pi\Delta t$.



Figure 5. Locus of a portion of the analytic signal obtained front sensor 1 on the complex plane.

Figure 6 shows the instantaneous frequencies from 0.3 s to 0.31 s, which clearly shows the average rotation frequency of 30 Hz (f_0), fluctuation amplitude of 1 Hz (Δf), and fluctuation frequency of 300 Hz (f_m).

The parameter c_b , is very important since it theoretically determines the upper frequency limit of the instantaneous rotation frequency spectrum. Since the widest sidelobes are at $f_Z(1\pm c_b)$, the highest modulation frequency is equal to $c_b f_Z$ (= $c_b f_0 Z$). This frequency will be referred to as the "highest analysis frequency." Then the highest order of analysis is given by $c_b Z$. As a gear with a larger number of teeth Z is used, the highest order can be increased.



Figure 6. Instantaneous rotation frequency within 0.01s time interval for the case with $f_m = 300$ and $\Delta f = 1$.

EXAMPLES OF NUMERICAL ANALYSIS BY THE PROPOSED METHOD

The rotation frequency and its fluctuations are usually analyzed both in the time domain and the frequency domain. In the former, results may be presented by time-dependent instantaneous frequencies. In the latter, they may be presented by power spectra as a function of frequency or magnitudes of frequency components as a result of the order analysis. The analysis accuracies of the proposed method will be evaluated both in the time and the frequency domains.

ANALYSIS PARAMETER SETTING

Before going into details of numerical analysis, the analysis parameter setting should be discussed. In the process of applying Eq. (5), the correction bandwidth on the frequency axis should be optimized. The selection of the correcton bandwidth is directly related to the divide-by-zero problem in applying Eq. (5). The lowest two frequencies at which $\{1-e^{-j2\pi kM/N}\}$ become zero are 0 and f_s/M . In the present study, a very small value, 0.000001, is added to the denominator in the analysis program so that the program will run at least without being interrupted by the divide-by-zero problem.

The lower cutoff frequency of the correction band is set equal to c_1f_0 with $c_1 = 0.5$. The parameter $c_1 = 0.5$ is chosen in order to let the lower cutoff frequency be below the possible lowest frequency, f_0 , of the spurious instantaneous rotation frequency components and to avoid the effect of the first divide-by-zero at frequency 0.

The higher cutoff frequency of the correction band, c_2f_Z , can be set to any value below $f_s/2$ in order to see the effect of the correction in the frequency domain (the program runs without being interrupted by the divide-by-zero problem). In the present study, $c_2 = 2$, will be used, which sets the highest correction frequency approximately 3 times higher than the highest analysis frequency (= $0.7 f_Z$). If the time waveform is required, two conditions to determine the higher cutoff frequency, c_3f_Z , must be considered. The first one is:

$$c_3 f_Z < c_b f_Z \tag{9}$$

In the present study, $c_3 = 0.65$ may be an appropriate choice. The second condition is that the higher cutoff frequency must be lower than the frequency of the second divide-by-zero with some margin, because the effect of the divide-by-zero is comparatively wide (see Fig. 18). In order for c_3f_Z to be lower than 75 % of the second zero, (f_s/M) , the following condition must be satisfied.

$$c_3 f_Z \le 0.75 (f_s / M)$$
 or $c_3 \le 0.75 (M_Z / M)$
(10)

The coefficient should be determined by $c_3 = 0.65$ or $c_3 = 0.75(M_z/M)$, whichever is lower. If the two sensors are separated with the condition,

$$M \le M_Z \tag{11}$$

the coefficient, $c_3 = 0.65$, satisfies the two conditions given by Eqs. (9) and (10).

In the following analysis, $f_0 = 30$, Z = 120, $f_s = 51,200$, T = 2, $N = 2^{18}$, $c_b = 0.7$, $c_1 = 0.5$, $c_2 = 2$, and $c_3 = 0.65$, will be used throughout the following sections if not otherwise stated. $N = 2^{18}$ (= 262,144) is used by adding 0 data to the pulse signals with 2×51200 (= $T \times f_s$) sample points in order to have a finer frequency resolution.

TIME DOMAIN ANALYSIS

The analysis result shown in Fig. 6 is for the ideal gear with $\Delta f = 0$ (without fluctuation) and M = 14 ($\approx M_Z$). In the case of the faulty gear, the instantaneous rotation frequencies contain spurious components. The two instantaneous frequencies obtained by sensors 1 and 2 of the faulty gear with the same condition ($\Delta f = 0$ and M = 14) are shown in Fig. 7 on the time axis from 1.022 to 1.025. The instantaneous frequencies largely deviate from 30 Hz by about 1.7 % when the 120th tooth comes under the sensing area of each sensor. If only one sensor is used, this fluctuation must be taken as a real rotation fluctuation since there is no way to distinguish this fluctuation from the true fluctuation.



Figure 7. Instataneous frequecies obtained from sensors 1 and 2 for the case with the faluty gear without rotation fluctuation before the correction with M = 14.

Figure 8 shows instantaneous rotation frequencies of pulse signals with $\Delta f = 0$ and M = 14 before (top) and after (bottom) the correction. The instantaneous rotation frequencies before the correction show spikes at every 1/30s. After the correction, those spikes are almost completely removed and the instantaneous frequencies remain within ± 0.00001 of f_0 in most of the time range. The extremely small error rate (0.00001) indicates the effectiveness of the proposed method.



Figure 8. Instantaneous frequecies of the faulty gear without rotation fluctuation before (top) and after (bottom) the correction with M = 14.

Figure 9 shows instantaneous frequecies of the faulty gear without rotation fluctuation after the correction for the cases with M = 7 and M = 28. The errors for M = 7 are slightly smaller than those for M = 14. The errors for M = 28 are much larger than M = 14 because the second divide-by-zero is within the range of analysis.



Figure 9. Instantaneous frequencies of the faulty gear without rotation fluctuation after the correction for M = 7 and 28.

Figure 10 shows instantaneous frequencies for the case with $f_{\rm m} = 300 \ (10^{\rm th} \text{ order}), \ \Delta f = 0.003$, and M = 14 in the range from t = 1.01 to 1.04. The fluctuation amplitude, $\Delta f = 0.003$, is chosen for analysis examples because it corresponds to the modulation depth $(d_{\rm m})$ of 0.0001. The left- and right-hand side vertical axes are for before and after the corrections. As expected from Fig. 8, the true rotation and fluctuation frequencies are obtained after the correction without being contaminated by the errors caused by the unevenness of the gear teeth and other errors.



Figure 10. Instataneous frequencies of the faulty gear before and after the correction ($f_m = 300$, $\Delta f = 0.003$, and M = 14).

The above examples (Figs. 8, 9, 10) are under the ideal condition-i.e., the sensors are separated exactly by integer numbers of sample points (M = 7, 14, and 28). M = 7 and M = 14 also satisfy the condition, Eq. (10), for the case with $c_3 = 0.65$. If M deviates from an integer, the accuracy is seriously deteriorated. Fig. 11 shows the case with $f_m = 300$, $\Delta f = 0.003$, and M = 14.5 in which large components with higher fluctuation frequencies seem to be added to the true instantaneous frequencies. A reason for this will be made clear in the next section.



Figure 11. Instataneous frequencies of the faulty gear before and after the correction ($f_m = 300$, $\Delta f = 0.003$, and M = 14.5).

Figure 12 shows the instantaneous frequencies of sensors 1 and 2 with $\Delta f = 0$ and M = 14 (top) and 14.5 (bottom). The instantaneous frequency obtained by sensor 1 is shifted backward by 14 samples for both M = 14 and 14.5. If M = 14, the two curves match almost perfectly. However, if M = 14.5, the shapes of the two curves slightly differ. This is because different positions of the gear are sampled by sensors 1 and 2. If the two instantaneous frequency waveforms are different with the condition $\Delta f = 0$, it is impossible to completely eliminate the error caused by the unevenness of the tooth shape.



Figure 12. Instantaneous frequencies obtained from sensors 1 and 2 of the faulty gear without fluctuation for M = 14 (top) and 14.5 (bottom). The curves of sensor 1 are shifted backward by 14 samples.

Results shown above indicate that the proposed method works extremely well for cases when M takes integer values but the effectiveness is decreased as M deviates from integer values. In this case, if a true fluctuation is very low, there is some limitation to separate it from spurious fluctuations in the time domain. Since it is not a simple task to let M take an integer value in real measurements, some way of avoiding this limitation should be derived. The frequency domain (spectrum) analysis will help to make the situation clearer, and it will also contribute to a quantitative evaluation of the accuracy of the proposed method.

FREQUENCY DOMAIN ANALYSIS

Figure 13 shows spectra of instantaneous frequencies of the ideal gear before and after the correction for the cases with M = 14 (top) and M = 14.5 (bottom). Other parameters are $f_m = 300$ and $\Delta f = 0.003$. The DC components of the instantaneous frequencies of sensors 1 and 2 are removed before the spectral analysis. The vertical axis is normalized so that 0 dB corresponds to $\Delta f = 1$. The fluctuation component at 300Hz remains unremoved without being affected by the correction for both cases. For the case with M = 14, other peaks, including the largest peak at 800 Hz, are significantly reduced by the proposed method. If M = 14.5, however, the peak level at 800 Hz increases by 3.6 dB after the correction. This is the cause of difficulty to identify the 300 Hz component in the time domain waveform (see Fig. 11).

It was shown that 400Hz and its integer multiple components exist in the spectrum of the rotation pulse for the case with f_0 = 30 and f_s = 51,200, and they may cause instantaneous frequency estimation errors [5].



Figure 13. Spectra of the instantaneous frequencies of the ideal gear before and after the correction for M = 14(top) and 14.5 (bottom). Other parameters are $f_m = 300$ and $\Delta f = 0.003$.

Figure 14 shows spectra of instantaneous frequencies of the faulty gear. Other parameters are the same with Fig. 13. Before the correction, integer order components with almost identical levels exist in both cases (M = 14 and 14.5). After the correction, they are reduced by more than 40 dB for both cases, indicating that the proposed method works well for any value of M to remove the errors caused by the unevenness of the tooth shape. For the case with M = 14, the peak at 800 Hz (and components around it) is also moved, but not for the case with M = 14.5. This means that the delay of this component between the two sensors is equal to 14 for M = 14

14, but different from 14.5 for M = 14.5. A detailed analysis shows that the phase of the 800 Hz component changes by π when M changes from 14 to 14.5. Since one sample point delay of an 800 Hz sinusoidal signal is equivalent to 0.098 rad, the above change by π is too large. If M is an integer, sensor 2 samples exactly the same points of the gear as sensor 1 does. If M deviates more from an integer, more distant points of the gear are sampled by sensors 1 and 2. It is inferred that this difference of sampling positions causes the large phase difference of 800 Hz components between sensors 1 and 2.

The sharp fall off of the spectrum before the correction indicated by the horizontal arrow in Fig. 14 (top) is the highest analysis frequency determined by $c_b f_Z$ (=2,520). The next peak indicated by the vertical arrow is due to the second divide-by-zero, whose frequency is given by f_s/M (=3,657). If $c_3 = 0.65$ is used, these two frequencies are excluded when the inverse Fourier transform is applied (see Eqs. (5), (10), and (11)).



Figure 14. Spectra of the instantaneous frequencies of the faulty gear before and after the correction for the case with M = 14 (top) and 14.5 (bottom). The other parameters are the same with those of Fig. 13.

<u>Figure 15</u> shows spectra of the instantaneous frequencies of the faulty gear with $f_s = 40,960$ and 102,400. Other conditions are the same as in <u>Fig. 14</u> (bottom). Two large peaks at 1360 and 2240Hz are observed, and a peak at 1600Hz is observed, in the top and bottom charts, respectively. <u>Figures 13</u>, <u>14</u>, <u>15</u> show that frequencies of the extraneous instataneous frequency components, that are different from those caused by the unevenness of the tooth shapes, are influenced in a complex manner by $M (= f_s/Z f_0)$). In general, as M increases, levels of extraneous components decrease.



Figure 15. Spectra of the instantaneous frequencies of the faulty gear before and after the correction for the case with $f_s = 40,960$ (M = 11.5, top) and 102,400 (M =28.5, bottom). Other conditions are the same as those of Fig. 13.

The dependences of the peak levels at 800 Hz on M in the case of faulty gear is shown in Fig. 16 in the range from M = 14 to 16. The figure shows that M must be very close to an integer for the peak level to be lower than -70 dB (effective fluctuation amplitude of approximately 0.001 Hz or modulation depth of approximately 0.0003). If a testing system has a function of very accurate rotation speed control, there is a possibility to set M within a desired range.



Figure 16. Dependence of peak levels at 800 Hz on M.

As mentioned above, the generation of the extraneous components is strongly influenced by *M*. If *M* varies in time, in other words, if there is a slowly fluctuating component, the extraneous component(s) may be dispersed. Figure 17 shows the case with two fluctuation components. The first one is with $f_{m,1} = 10$ and $\Delta f_{m,1} = 0.3$, and the second one is with $f_{m,2} = 300$ and $\Delta f_{m,2} = 0.003$. The extraneous component at 800 Hz has disappeared or at least it has become invisible. In real situations, since it is very common that there are slowly fluctuating components with relatively large fluctuation amplitudes, the extraneous components may not be a big problem compared to those seen in Figs. 13 (bottom), 14 (bottom), and 15 (top).

Since the levels and frequencies of the extraneous components depend strongly on M (or f_s), measurements should be done with two sampling frequencies at the beginning of testing. If levels of peaks of interest are not affected by f_s , those are considered more reliable.



Figure 17. Spectra of the instantaneous frequencies of the faulty gear before and after the correction for the case with $f_{m,1} = 10$ ($\Delta f_1 = 0.3$), $f_{m,2} = 300$ ($\Delta f_2 = 0.003$), and M = 14.5.

EXAMPLE OF EXPERIMENTAL ANALYSIS BY THE PROPOSED METHOD

One application example of the proposed method to a flywheel ring gear in a timing gear train of a diesel engine is described in this section. Two eddy-current type sensors were attached to the ring gear which has 160 teeth. The sensors had to be separated by a width of approximately 3.5 teeth because of an interaction between the sensors (this problem can be solved by using two different carrier frequencies in the sensing circuits). The rotating speeds tested were 475 rpm (f_0 = 7.92) and 1530 rpm (f_0 = 25.5). Other measurement and analysis parameters were Z = 160, $f_s = 25,600$, T = 1.263 ($f_0 = 7.92$) and 0.392 ($f_0 = 25.5$), $N = 2^{15}$, $c_b = 0.7$, $c_1 = 0.5$, and $c_2 = 2$.

Results for the 475 rpm (top) an 1530 rpm (bottom) are shown in Figure 18. The vertical axes are in modulation depth, whose reference level (0dB) is $d_m = 1$. The horizontal axes are in Hertz, whose scales are adjusted by a factor of 25.5/7.92 for the ease of comparison. The results clearly show that spurious components are removed by the correction. Since the patterns and levels of the spurious components are very similar for the two rotation frequencies, it seems reasonable to consider that those are caused by the unevenness of tooth shapes of the gear. The spurious component with the highest level (indicated by the arrows in the figure) corresponds to the 40th order, which may be related to the number of teeth, Z = 160. The level difference between the highest true fluctuation component (3rd order in both cases) to the highest spurious component before the correction are 30.1 and 21.1 dB for $f_0 =$ 7.92 and 25.5, respectively. These numbers are increased to 44.3 and 36 dB after the correction. Since the spurious components are widely distributed, the time waveforms before and after the correction may be more different than expected from the spectral level differences.

Effects of the divide-by-zero are clearly seen at around 364 and 1163 Hz for $f_0 = 7.92$ and 25.5, respectively. The measured M/M_z is 3.49 (= 70.68/20.2) and 3.51 (= 21.99/6.27) for $f_0 = 7.92$ and 25.5, respectively. The frequencies agree with those given by f_s/M . If the sensor separation is reduced close to M_Z , the effects of the divideby-zero will be shifted outside of the highest analysis frequency (887 Hz and 2,856 Hz, respectively) and the improvements of the correction will increase. The waveform in the time domain is not shown here because M/M_z is too large (see Eq. (11)).



Figure 18. Measured spectra of rotation fluctuations of a flywheel ring gear used for a diesel engine before and after the corrections. The vertical axes are in modulation depth (reference level is $d_m = 1$).

SUMMARY

A method of removing rotation frequency estimation errors caused by the unevenness of tooth shapes used for the measurement of rotation fluctuations was proposed. The main feature of the method is to use two sensors instead of one sensor as in the conventional method. Two instantaneous rotation frequencies as functions of time are obtained from the two sensors, which include true and erroneous instantaneous frequency components. The subtraction of those two cancels out the true fluctuation since they are included in the same form in the total instantaneous frequency functions. The remaining is a combination of a spurious instantaneous frequency function and its delayed version with the delay equivalent to the separation angle between the two sensors. These two can be separated in the frequency domain by applying the discrete Fourier transform. If the error function is separated, it can be subtracted from the instantaneous frequency functions of sensors 1 or 2.

The validity of the proposed method was shown by numerical examples. In applying the proposed method, the relations of several parameters for the analysis were made clear. A rotation pulse generation model was used for the generation of rotation pulse signals with a known rotation speed and fluctuations. This was essential because the results were compared directly with the known fluctuations. One analysis example showed that the accuracy was on the order of 0.00001 when expressed in terms of the modulation depth in an ideal condition. Other causes of errors and how to avoid them were also discussed.

The proposed method was applied to an actual measurement, the results of which proved the effectiveness of the method in practical applications, even though the two sensors were not positioned as closely to each other as desired.

REFERENCES

1. Ishihara, Akira, Sugito, Hiroshi, and Nishiyama, Toshihiko, A measurement techniques for the rotational movement of gear trains using Hilbert Transform, *Proceedings of JSAE Annual Congress*, 952, p.105-108 (1995-5).

2. Ohara, Kouzi, Kitagawa, Fukuo, Yaoko, Seiji, and Shimizu, Kouichi, A New Measurement Method for the Rotational Movement of Gear under Transient State-The Application to Analyze Gear Noise during Acceleration, *Proceedings of JSAE Annual Congress*, 117-02, p.5-8 (2002-5).

3. Yamaguchi, Hiroki and Yamaura, Hiroki, Gear Rattle Analysis: The signal processing for mesh model and gear behavior, *Proceedings of JSAE Annual Congress*, 11-08, p. 5-10 (2008-5).

4. Nakasima, Takashi, Sugisaki, Makoto, and Suzuki, Hideo, Instantaneous Rotation Frequency/Phase Measurement of a Rotor by the Analytic Signal Method-Analysis Precision Study Using a Rotation Detector Model, *Proceedings of JSAE Annual Congress*, 20-09, p. 1-6 (2009-5).

5. Nakashima, Takashi, Sugisaki, Makoto, Tatekawa, Hirokazu, Mizukawa, Hisanobu, and Suzuki, Hideo, Analysis Accuracies of Frequency Fluctuations of Rotation Pulse Signals by the Analytic Signal Method, *Proceedings of JSAE Annual Congress*, 14-10, p.13-18 (2010-5).

6. Suzuki, Hideo, Ma, Furong, Izumi, Hideaki, Yamazaki, Osamu, Okawa, Shigeki and Kido, Ken'iti, Instantaneous frequencies of signals obtained by the analytic signal method, Acoustical Science & Technology: 27, 3, p. 163-170 (2006).

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DEFINITIONS/ABBREVIATIONS

φ (n)

instantaneous phase (radian)

Δf

0-p amplitude of frequency modulation (Hz)

Δt

sampling period (=1/fs) (s)

c_i

coefficients to determine cutoff frequencies (i=1,2,3)

c_b

filter bandwidth constant

d_m

modulation depth $(=\Delta f/f_0)$

f₀

(average) rotation frequency of the gear (Hz)

$f(n), f_i(n)$

instantaneous rotation frequencies (i = 1, 2)

f_m

frequency of frequency modulation (Hz)

f_s

sampling frequency (Hz)

$f_{sp}(n)$

spurious instantaneous rotation frequency (Hz)

$F_{sp}(k)$

Fourier transform of $f_{sp}(n)$

fz

tooth frequency $(=f_0 \times Z)$ (Hz)

$f_{tr}(n)$

true instantaneous rotation frequency (Hz)

k

number representing discrete frequencies

М

number of sample points that corresponds to the angle between the sensors

M_Z

n

N

T

number of sample points that corresponds to the one tooth angle $(=f_S/(f_0 Z))$

number representing discrete times $(t = n\Delta t)$

size of FFT and IFFT

pulse signal length (s)

$x_i(n)$

rotation pulse signal (i=1, 2)

Ζ

number of teeth of the gear

2

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