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## Comparison of FRF Curve Fitting Methods for the Loss Factor Measurement

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### Abstract

The half-power-width method is the standard method for the loss factor measurement found in documents of ISO and other standard organizations. However, this method cannot be used for materials with large loss factors with a high degree of modal overlap. The curve fitting method used for the modal analysis is another choice. It is known, however, that this method does not have satisfying fitting capabilities when applied to real frequency response functions (FRF) for the purpose of loss factor measurement. In order to improve the fitting performance, a curve fitting method based on the pole-zero model was proposed. In order to fit troughs as well as peaks of FRF's, two types of error functions were also proposed. In this paper, results of fitting of model FRF's to measured FRF's with the conventional method and the proposed method are compared. It is concluded that the proposed method has better fitting performance compared to the conventional method.

### 1. Introduction

Accurate and efficient evaluation methods of the loss factor of materials are crucial for the development as well as for the application of materials. The half-power width method has been the main method for this purpose. After the experimental modal analysis became popular, the curve fitting technique has been added as one of the methods of the loss factor measurement. However, it seems to be the general understanding among experts group that the curve fitting method used for the modal analysis is not good enough for the purpose of loss factor measurement. There are two

possible reasons experts think that way. One is that they are interested in the "loss factor" of a material at its anti-resonances as well as at its resonances. If it is possible to measure the loss factor at the anti-resonances, the measurement becomes more efficient because they can obtain more data at different frequencies from one FRF measurement. The measurement (assuming that it is meaningful) is easier at the anti-resonances than at the resonances because the ratio of the effective mass of the measurement system to the impedance of the test piece is smaller at the anti-resonances. Another reason is related to the first one but more fundamental. The most common physical model of the driving point mobility used in the modal analysis is equivalent to the electrical circuit representing the parallel connection of resonance circuits with L (inductance), C (capacitance), and R (resistance). Obviously, this equivalent electrical circuit model can represent the resonance characteristics (peaks of the mobility). On the other hand, if this model is once employed, one must accept the fact that the anti-resonance characteristics are determined from those of the resonance characteristics. Since we do not know the relation between the loss factors at the resonances and anti-resonances of an actual material, we must use a pole-zero model which allows us to represent both the resonance and anti-resonance characteristics independently [1]. Details of the curve fitting method using the pole-zero model are described in reference [2]. In this paper, fitting performances of the conventional and the proposed methods are compared by applying those methods to measured FRF data. Results show that the proposed methods give better fitting in general than the conventional method.

## 2. FRF Models

### 2.1 FRF Model Used for Modal Analysis

The commonly used "mobility" model in the field of modal analysis is given by

$$X(\omega) = \sum_{n=1}^N \frac{(j\omega / m_n)}{[-\omega^2 + j(\omega_n \eta_n)\omega + \omega_n^2]} + \frac{1}{j\omega m_0} + \frac{j\omega}{k_0} \quad (1)$$

where  $\omega_n$ ,  $\eta_n (=c_n/(m_n k_n)^{1/2})$ , and  $m_n$  are the resonance frequency, loss factor, and mass of the  $n$ -th mode, respectively. An electrical circuit equivalent to Eq.(1) based on the impedance analogy is given by Fig.1, where  $c_n$  and  $k_n$  are the damping coefficient and the spring constant of the  $n$ -th mode.

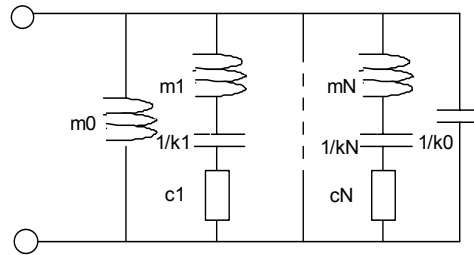


Fig.1 Equivalent electrical circuit of the conventional FRF model used in the modal analysis

Each branch with  $m_n$ ,  $1/k_n$ , and  $c_n$ , represents the  $n$ -th mode resonance. The anti-resonance characteristics result from the parallel connections of the adjacent resonant branches. This type of model does not have a freedom to independently define both the resonance and the anti-resonance characteristics. Eq.(1) or Fig.1 is a resonance-based model. The driving point FRF can be represented by an anti-resonance based model using the same impedance analogy. They are identical for a loss-less system, but they differ when the damping is introduced to the system [1].

## 2.2 Pole-zero Model

Another approach to the modeling of the driving point FRF is to start from the pole-zero model considering that all the poles and zeros are independent. For example, the driving point mobility of the free-free beam is given by

$$X_{A\pm}(\omega) = \frac{H(\omega_1^2 - \omega^2 + j\omega_1\eta_1\omega) \cdot \dots \cdot (\omega_{2N\pm 1}^2 - \omega^2 + j\omega_{2N\pm 1}\eta_{2N\pm 1}\omega)}{j\omega(\omega_2^2 - \omega^2 + j\omega_2\eta_2\omega) \cdot \dots \cdot (\omega_{2N}^2 - \omega^2 + j\omega_{2N}\eta_{2N}\omega)} \quad (2)$$

where  $H$  is a constant. The sign  $+$  or  $-$  represents whether the model has a pole or zero at the infinite frequency.  $X_{A+}$  is equivalent to Eq.(1) since both have a pole at the infinite frequency. A significant difference between Eq.(1) and Eq.(2) is that, if we start from Eq.(2), poles and zeros can be defined independently. Since we must fit a measured FRF with the model FRF at both the resonances and anti-resonances, it is necessary to use a model of the type of Eq.(2).

## 3. Error Functions and Curve Fitting

### 3.1 Conventional Error Function

The general method of curve fitting is to minimize the error function  $\lambda$  defined below.

$$\lambda = \sum_{m=1}^M |A(\omega_m) - X(\omega_m)|^2 \quad (3)$$

where  $A(\omega_m)$  and  $X(\omega_m)$  are the measured and model FRFs, respectively, and  $\omega_m$  is the angular frequency used for the fitting. This type of error function cannot be used in order to curve fit both the resonance and anti-resonance characteristics, because errors around the peaks (poles) of FRF contribute more to the error function and consequently the fitting around zeros may be worse percentage-wise. In order to solve this problem, two error functions were proposed [2].

### 3.2 Normalized Error Function

The normalized error function is the one in which the squared absolute of the difference between the measured and model FRF's is normalized to the power of the measured FRF.

$$\lambda_{\text{norm}} = \sum_{m=1}^M |A(\omega_m) - X(\omega_m)|^2 / |A(\omega_m)|^2 \quad (4)$$

By the above normalization, the contribution of each term becomes level-independent.

### 3.3 Decibel Error Function

The following error function  $\lambda_{dB}$  is used in the "decibel method."

$$\lambda_{dB} = \sum_{m=1}^M |A_{dB}(\omega_m) - X_{dB}(\omega_m)|^2 \quad (5)$$

where  $A_{dB}(\omega_m)$  and  $X_{dB}(\omega_m)$  are the measured and model FRFs in decibel, respectively. By taking the differences in decibel, the model FRF should be fitted to the measured FRF independently on the levels of the measured FRF at individual frequencies.

### 3.4 Curve Fitting Methods

The common method by which the error function is minimized in the frequency domain is the differential iteration method. Please see reference [2] or modal analysis books for details. The methods used here are based on the differential iteration method with the combination of the models and the error functions. Those are shown in Table 1. For example method "C" is the one in which the pole-zero model and the normalized error function are used.

Table 1. Four methods used for the evaluation of the fitting performance.

	conventional error func.	normalized error func.	decibel error func.
conventional model	A	B	
pole-zero model		C	D

## 4. Tested Materials

The materials used for the evaluation of the curve fitting are shown in Table 2. The first two are metals and the latter two are resins.

Table 2. Materials used for the evaluation of the curve fitting methods

symbols	general naming	description
M2052	damping alloy	Mn-20Cu-5Ni-2Fe
Fe+LR	unconstrained type damping plate	steel base plate (1mm thickness) + damping polymer sheet (2mm thickness)
ABS	general purpose resin	acrylonitrile-butadiene-styrene
PP	general purpose resin	polypropylene

## 5. Results and Concluding Remarks

FRF's of the materials shown in Table 2 were measured by driving the center of rectangular test pieces with the free-free boundary. Those are shown by the thin (black) lines in Figs.2-5. Thick (pink) lines in the figures are model FRF's after the curve fitting. Curves near the bottom of each

charts show the difference in decibel between the measured and model FRF's after the curve-fitting.

Following (concluding) remarks can be made from the results.

- 1) Method A is the worst among four methods.
- 2) Method B shows a significant improvement from method A.
- 3) Method C and D are better than the methods A and B.
- 4) Method D shows slightly better performance than method C.

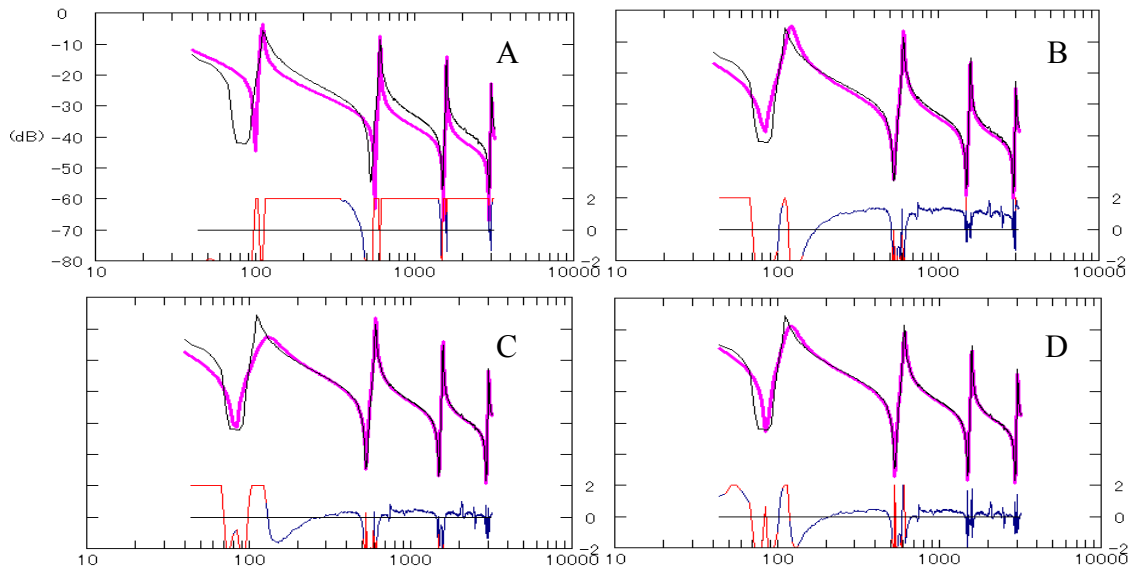


Fig.2 Fitting results for M2052 with methods A-D.

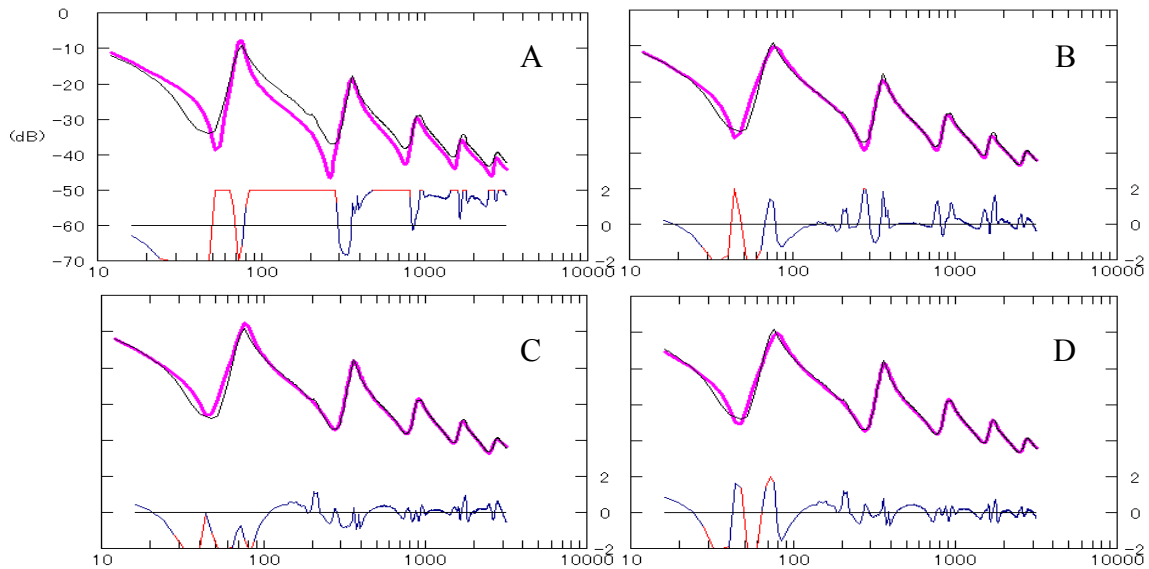


Fig.3 Fitting results for Fe+LR with methods A-D.

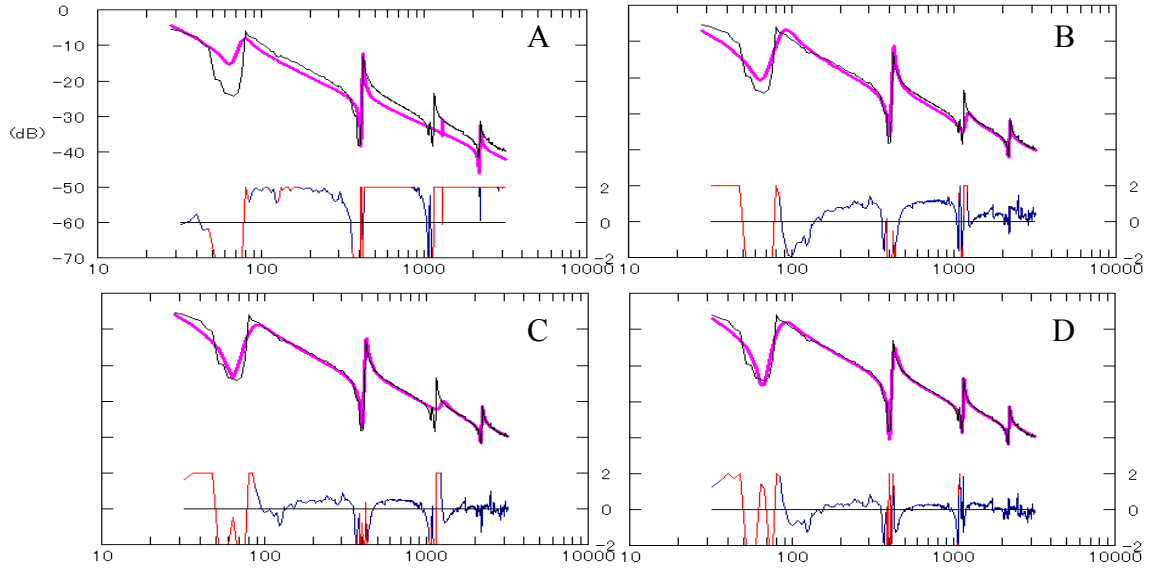


Fig.4 Fitting results for ABS with methods A-D.

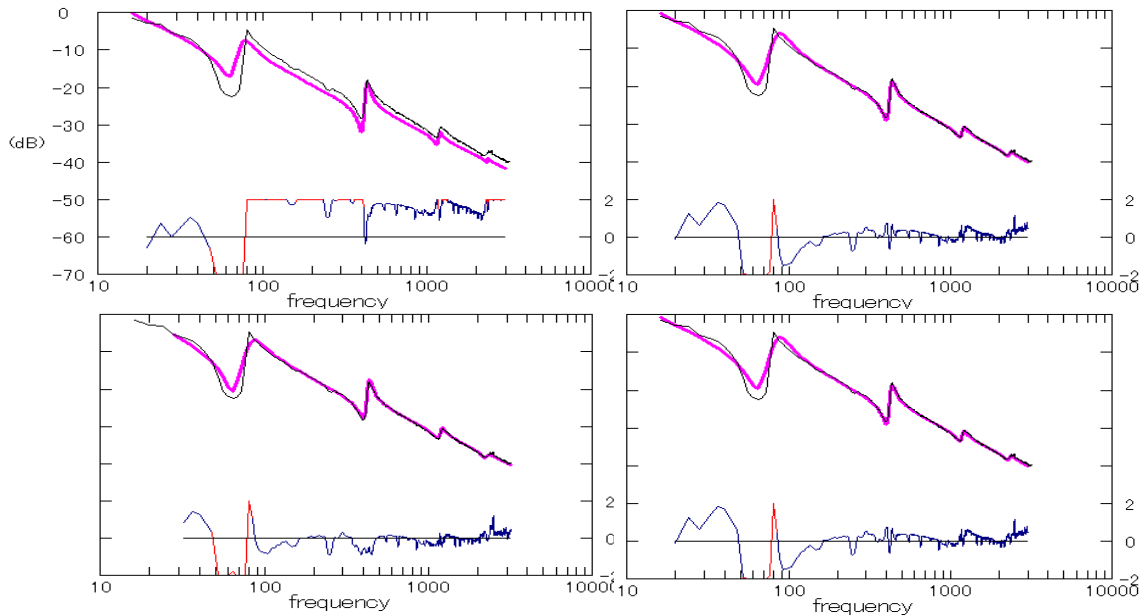


Fig.5 Fitting results for PP with methods A-D.

## References

1. H. Suzuki, K. Kido, and T. Ono, Equivalent electrical models used for the loss factor measurement. *J.Acoust.Soc.Jpn.(J)* **57**, 265-271, 2001.
2. H. Suzuki, S. Nakazawa, T. Ono, and K. Kido, Curve fitting of frequency response functions by use of pole-zero models for the loss factor measurement, *Acoust.Sci & Tech.* **22**, 344-350, 2001.