

Self and mutual radiation resistance measurement of two closely located loudspeakers

Hideo Suzuki,* Masazou Anzai,* Takahiko Ono,* Frank Fossaert**

**Ono Sokki Co., Ltd.,*

1-16-1, Hakusan, Midori-ku, Yokohama, 226 Japan

***Institut Supérieur D'Electronique de Paris,*

Paris, France

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In an active noise control or in sound reproduction, it is important to know self and mutual radiation impedances of sound sources. Since the total radiated power from sound sources is determined by self and mutual radiation resistances and velocities of the radiating surfaces, it is possible to determine the radiation resistances from the measurement of radiated power and velocities. Radiation resistances of two loudspeakers measured in a large meeting room are on the same order of a rigid piston in an infinite baffle, but show large irregularities due to complex radiation conditions. The mutual radiation resistance decreases significantly by increasing the distance between the loudspeakers.

Keywords: Loudspeaker, Sound radiation, Radiation impedance, Self radiation resistance, Mutual radiation resistance, Sound intensity

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1. INTRODUCTION

It is possible to theoretically calculate self and mutual radiation impedances of a very simple sound source model, for example, a rigid piston in an infinite baffle.¹⁾ However, those of a source in a real environment is almost impossible to calculate. In this paper, a method of obtaining radiation resistances of a sound source in a real environment is proposed.

For a two-loudspeaker source, the unknown parameters that determine total radiated power are two self radiation resistances and one mutual radiation resistance, R_{11} , R_{22} and R_{12} , respectively. If we can measure radiated power W_i for three different drive conditions ($i=1, 2,$ and 3) and diaphragm velocities, we can determine the above three unknowns. If once R_{11} , R_{22} and R_{12} are known and if a vibration of one speaker is fixed, it is possible to control so that the total radiated power may be

minimum by selecting the amplitude and phase of the other speaker.

The present paper proposes at first a method used to obtain radiation resistances from measurements of radiated power and diaphragm velocities of a two-loudspeaker source. A sound intensity method used to measure the radiated power is also reviewed briefly. Then, results of three experiments at different locations in a large meeting room, are reported. A comparison of these results makes it possible to evaluate the influence of the environment upon radiation characteristics of sources.

2. THEORY

2.1 Self and Mutual Radiation Impedances and Radiated Power of a Two-Loudspeaker Source

Forces F_1 and F_2 acting on two rigid discs, are given, respectively, by²⁾

$$F_1 = Z_{11}V_1 + Z_{12}V_2, \quad (1)$$

and

$$F_2 = Z_{21}V_1 + Z_{22}V_2, \quad (2)$$

where Z_{11} and Z_{22} are the self-radiation impedances and Z_{12} ($=Z_{21}$) is the mutual radiation impedance. The total radiation impedance Z_1 and Z_2 of the discs I and II are given, respectively, by

$$Z_1 = F_1/V_1 = Z_{11} + Z_{12}(V_2/V_1), \quad (3)$$

and

$$Z_2 = F_2/V_2 = Z_{22} + Z_{12}(V_1/V_2). \quad (4)$$

The individual power radiated by disks I and II are given, respectively, by

$$(1/2)F_1V_1^* = (1/2)Z_{11}|V_1|^2 + (1/2)Z_{12}V_1^*V_2 \quad (5)$$

and

$$(1/2)F_2V_2^* = (1/2)Z_{22}|V_2|^2 + (1/2)Z_{21}V_1V_2^* \quad (6)$$

where the asterisk denotes a complex conjugate. The total complex power is then given by

$$W_c = (1/2)Z_{11}|V_1|^2 + (1/2)Z_{22}|V_2|^2 + Z_{12}|V_1V_2|\cos(\theta) \quad (7)$$

where $| \quad |$ denotes an absolute value and θ is the phase difference between V_2 and V_1 ($\arg(V_2/V_1)$). The effective power is given by taking a real part of W_c such that

$$W = (1/2)R_{11}|V_1|^2 + (1/2)R_{22}|V_2|^2 + R_{12}|V_1V_2|\cos(\theta) \quad (8)$$

where R_{11} , R_{22} and R_{12} are real parts (resistances) of Z_{11} , Z_{22} , and Z_{12} respectively. The last term of Eq. (8) is the power due to the acoustical coupling between the disks. If $\theta = \pm 90^\circ$, this term becomes zero and there is no coupling in terms of the total power. It should be noted, however, that there is an acoustical coupling as long as $|V_1V_2| \neq 0$ even if $\theta = \pm 90^\circ$, since the second term of Eq. (5) or (6) never becomes zero unless $Z_{12} = 0$. This means that one disk must radiate more energy than the other to maintain the same output.

2.2 Method of Determining Self and Mutual Radiation Resistances

At least three independent conditions are necessary in order to determine R_{11} , R_{22} and R_{12} . From three independent measurements, following three equations are obtained,

$$W_i = (1/2)R_{11}|V_{1i}|^2 + (1/2)R_{22}|V_{2i}|^2 + R_{12}|V_{1i}V_{2i}|\cos(\theta_i) \quad (i=1, 2 \text{ and } 3) \quad (9)$$

where i represents the number of measurement conditions.

Using a matrix notation, Eq. (9) is rewritten as

$$AB = C \quad (10)$$

with

$$A = \begin{bmatrix} (1/2)|V_{11}|^2 & (1/2)|V_{21}|^2 & |V_{11}V_{21}|\cos(\theta_1) \\ (1/2)|V_{12}|^2 & (1/2)|V_{22}|^2 & |V_{12}V_{22}|\cos(\theta_2) \\ (1/2)|V_{13}|^2 & (1/2)|V_{23}|^2 & |V_{13}V_{23}|\cos(\theta_3) \end{bmatrix}$$

$$B = \begin{Bmatrix} R_{11} \\ R_{22} \\ R_{12} \end{Bmatrix} \quad \text{and} \quad C = \begin{Bmatrix} W_1 \\ W_2 \\ W_3 \end{Bmatrix}$$

If $\det(A) \neq 0$ one can solve a set of simultaneous equations and determine R_{11} , R_{22} and R_{12} .

2.3 Minimizing Radiated Power by an Active Control

Let us consider one speaker as an original source of noise. In an active noise control, another source is added to reduce the total radiated power. With an assumption that the original source (disc I) is not influenced by the drive condition of the additional source (disc II), the drive condition for the additional source to minimize the total radiated power is found by differentiating Eq. (8) with respect to V_2 and θ ,

$$|V_1V_2|\sin(\theta) = 0 \quad (11)$$

and

$$R_{22}|V_2| + R_{12}|V_1|\cos(\theta) = 0 \quad (12)$$

Equations (11) and (12) give

$$\theta = 0 \quad (R_{12} < 0) \quad \text{or} \quad \theta = \pi \quad (R_{12} > 0) \quad (13)$$

and

$$|V_2| = |V_1||R_{12}/R_{22}|. \quad (14)$$

Rewriting Eq. (8) with these conditions, the total radiated power is given by

$$W = (1/2)R_{11}|V_1|^2 - (1/2)R_{22}|V_2|^2 \quad (15)$$

If once R_{11} , R_{22} and R_{12} are known, it becomes possible to control so that the total radiated power would be minimum by selecting the amplitude and phase of the additional source as given by Eqs. (13) and (14).

3. SOUND INTENSITY TECHNIQUE

The radiated power and the velocities of the source must be measured in order to experimentally determine R_{11} , R_{22} and R_{12} . A two-microphone sound intensity measurement technique is used for the measurement of radiated power. This technique is briefly described here.³⁾

A pressure and particle velocity including phase difference between them must be measured to obtain the intensity at a measurement point. Since a convenient high-accuracy particle velocity/voltage transducer is not available, the particle velocity is normally obtained indirectly from pressure measurement. Two pressure microphones are placed with separation d . Then, the pressure at the center of the two microphones is given in the frequency domain by

$$P(f) = [P_1(f) + P_2(f)]/2 \quad (16)$$

where $P_1(f)$ and $P_2(f)$ are Fourier transforms of pressure signals at microphone 1 and 2, respectively, and f is the frequency. The particle velocity in the direction from microphone 1 to 2 (i.e. in the measurement direction) is also given by

$$V(f) = -(1/j2\pi f\rho d)[P_2(f) - P_1(f)] \quad (17)$$

where j is the imaginary unit and ρ is the air density. A time-averaged sound intensity spectrum $I_a(f)$ is given as the real part of the crosspower spectrum G_{pv} between the pressure and the particle velocity as

$$I_a(f) = \text{Re}[G_{pv}(f)] \\ = -(1/2\pi f\rho d) \text{Im}[G_{p_1p_2}] \quad (18)$$

where $G_{p_1p_2}$ is the crosspower spectrum of the two microphone signals, which is easily calculated by use of the FFT (Fast Fourier transform) technique. The radiated power spectrum through surface S is given by

$$W(f) = \int_S I_{an}(f) dS \quad (19)$$

where $I_{an}(f)$ is the normal component of $I_a(f)$ to the measurement surface.

4. MEASUREMENT

4.1 Measurement Setup

The self and mutual radiation resistances of two loudspeakers (Technics SB-M5) were measured. Figure 1 shows the arrangement of the loudspeakers and measurement surfaces from S_1 to S_5 . The two loudspeakers, when they are put together, have the dimensions of 605 h \times 760 w \times 295 d. In the first two experiments, the two loudspeakers were placed as close as possible (380 mm from center-to-center). In the third experiment, the distance between the loudspeakers was increased by 150 mm. A block diagram of the measurement devices is shown in Fig. 2. A sound intensity analyzer (Ono Sokki SI-100) was used for the sound intensity measurement. In order to measure the radiation resistances of the

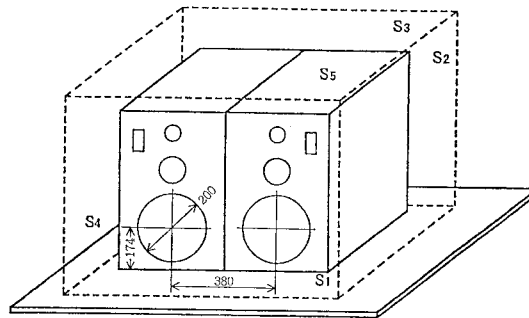


Fig. 1 Closely located loudspeakers and measurement surfaces.

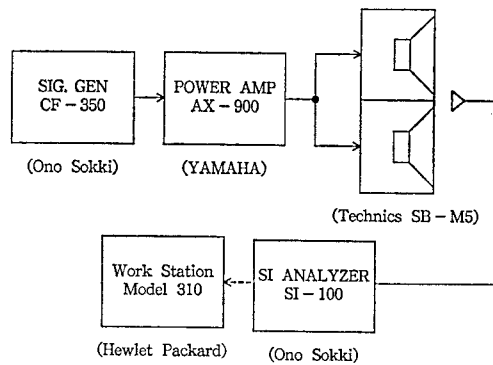


Fig. 2 Measurement setup.

low frequency units with a 200 mm diameter, only the low frequency units were driven. The upper limit of the measurement frequency range was 800 Hz. An FFT analyzer (Ono Sokki CF-350) was used as a signal generator to produce a pseudo-random signal.

4.2 Drive Conditions

It was shown in section 2.2 that measurements for at least three independent drive conditions enables to calculate the three radiation resistances. The first condition was to drive the two loudspeakers with the same voltage and phase. In this case, V_1 and V_2 were almost the same. The second condition was to drive the loudspeakers with the same voltage but with a reversed phase. Finally, the third condition was to drive only the left loudspeaker. The above three conditions will be referred to as conditions 1, 2, and 3, respectively. Phase differences for the first two conditions are shown in Fig. 3. The velocity spectrum of the two diaphragms when

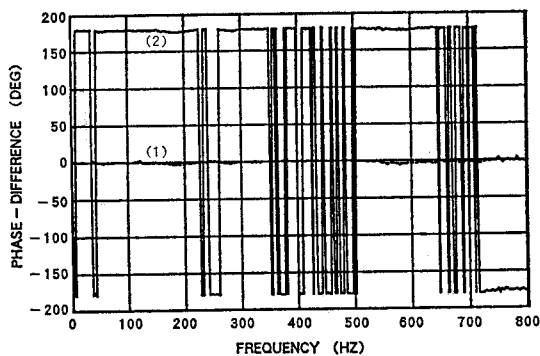


Fig. 3 Phase differences between the two speakers when they are driven in phase (1) and with a reversed phase (2).

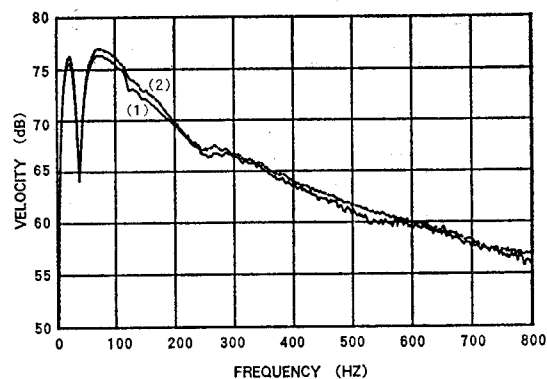


Fig. 4 Velocities of the left (1) and right (2) diaphragms when the two loudspeakers are driven in-phase and with the same voltage (ref. level = 10^{-9} m/s).

the two speakers are in phase is shown in Fig. 4. The velocity spectrum shown in Fig. 4 is normalized to 10^{-9} m/s. It was confirmed that the velocity spectrum showed only a slight dependence on the phase and on the location of the loudspeakers.

4.3 Loudspeaker Locations

Dimensions of the room used for the experiment and loudspeaker locations are shown in Fig. 5. At first we located the two loudspeakers closely to each other and not very close to any of the walls or ceiling. This will be referred to as experiment I (location 1). Then, the two loudspeakers were moved to a corner of the room to see the influence of the walls (experiment II at location 2). Finally, the loudspeakers

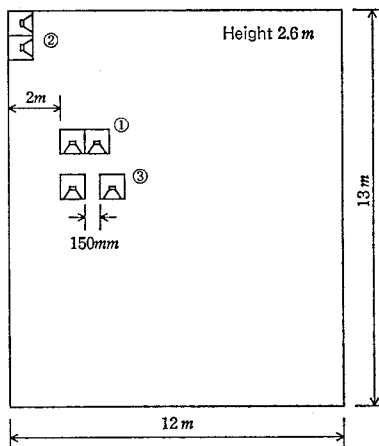


Fig. 5 Loudspeaker locations 1, 2, and 3 in a meeting room.

were placed at the same place as experiment I but with an increased distance between them (530 mm center-to-center, experiment III at location 1).

4.4 Microphone Probe Positioning

There are two ways of moving a microphone probe over a measurement surface. A method tried first was to move the probe point by point (discrete sampling). However, this method was too time consuming and, therefore, a continuous scanning method was tried next. Many times of averaging were carried out while the probe was moved continuously on each measurement surfaces. It was not very easy to move the probe parallel to the enclosure surfaces and at a constant speed. However, the result was in good agreement with that of the discrete sampling. This scanning method reduced the measurement time significantly and, therefore, it was employed through the whole experiment.

5. RESULTS AND DISCUSSION

5.1 Experiment I

The radiated power characteristics when the two loudspeakers are placed at location 1 are shown in Fig. 6. Figure 6 shows that the power radiated from the front surface is dominant in determining the total radiated power. Radiated power characteristics for three drive conditions are shown in Fig. 7. The power for condition 2 is more than 10 dB lower than the power for condition 1 below 200 Hz. However, the levels for condition 1 and 2 are re-

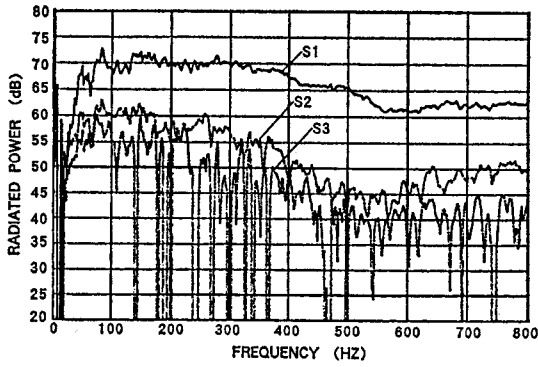


Fig. 6 Radiated power through S1, S2 and S3 when the two loudspeakers are driven in-phase and with the same voltage and placed at location 1 (ref. level= 10^{-12} W/m²).

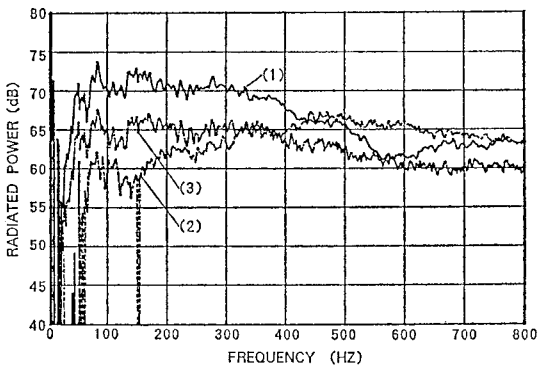


Fig. 7 Total radiated power characteristics for conditions 1, 2, and 3, when the two loudspeakers are placed at location 1 (ref. level= 10^{-12} W/m²).

versed above 450 Hz, indicating that the mutual radiation resistance changes its sign at around 450 Hz. The self and mutual radiation resistances of the loudspeakers at location 1 is shown in Fig. 8. They are normalized to $\rho c \pi a^2$, where c is the velocity of sound and a is the radius of the speaker. Figures 7 and 8 show that the sign of $(W_1 - W_2)$ corresponds to the sign of R_{12} . Since V_1 is almost the same for all drive conditions, Eq. (8) gives the following relations:

$$W_1 - W_2 = 2R_{12}|V_{11}V_{21}| \quad (20)$$

So, $W_1 > W_2$ if $2R_{12} > 0$ and $W_1 < W_2$ if $R_{12} < 0$.

It is seen in Fig. 8 that from 430 Hz the mutual

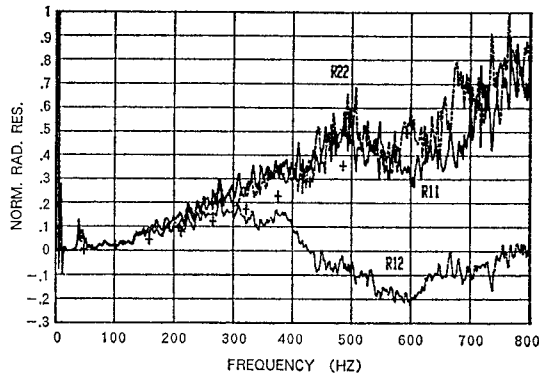


Fig. 8 Normalized self (R_{11} and R_{22}) and mutual (R_{12}) radiation resistances when the two loudspeakers are placed at location 1 (normalization factor is $\rho c \pi a^2$).

radiation resistance becomes negative. The mutual radiation resistance of two small spherical sources is given by

$$R_{12} = \pi a^2 c k^2 a^2 \sin(kx)/(kx) \quad (21)$$

where k is the wavenumber and x is the distance between them. For $x=380$ mm, R_{12} changes its sign when

$$kx = \pi \quad (22)$$

or

$$f = c/2x = 451 \text{ Hz} \quad (23)$$

The result of Fig. 8 agrees with that of this simple model.

Figure 8 shows that R_{11} is quite similar to R_{22} (this is expected because the conditions are similar for the two loudspeakers). Figure 8 also shows the self radiation resistance of a piston mounted in an infinite baffle which is indicated by “+.” The measured and calculated results show a similar trend. The differences may be due to the fact that the floor and the walls are working as reflecting surfaces and that the two loudspeakers are finite in size. In this case the real radiation condition is much more complex than the case of the rigid piston in an infinite baffle. A peak at approximately 40 Hz is because of the type of the loudspeaker (phase-inverted type). Figure 4 shows that the velocities are very small at this frequency. In order to get a correct value at 40 Hz, the effect of the enclosure openings must be taken into account as radiators.

As seen in Fig. 7 there is a 3 dB difference between

W_1 (or W_2) and W_3 at frequencies where R_{12} becomes zero. This is because only one channel is driven and roughly only the half power is radiated for W_3 .

5.2 Experiment II

Figure 9 shows the radiation resistances when the two loudspeakers are at location 2. A comparison of Fig. 9 to Fig. 8 enables one to see the influence of the environment upon the radiation resistances. One can see that at low frequencies R_{11} for experiment II is larger than that of experiment I. This is due to an increased number of image sources at the corner position than at other positions. The increased irregularity of Fig. 9 may be also due to the complex reflections at the corner position.

5.3 Experiment III

In the third experiment, the loudspeaker separation (placed at approximately the same position as experiment I) were increased by 150 mm. The results are shown in Fig. 10. The zero-cross frequency of the mutual radiation resistance is shifted lower compared to Fig. 8. Inserting $x=(0.380+0.150)$ into Eq. (23), zero-cross frequency is estimated at 324 Hz. The validity of this estimation is confirmed by Fig. 10. Another difference between Fig. 8 and Fig. 10 is that the maximum value of the mutual radiation resistance of Fig. 10 in the low frequency region (below 400 Hz) is approximately half of that of Fig. 8 indicating that the distance between the two sources has significant effects on the mutual radiation resistance. For a purpose of active noise control,

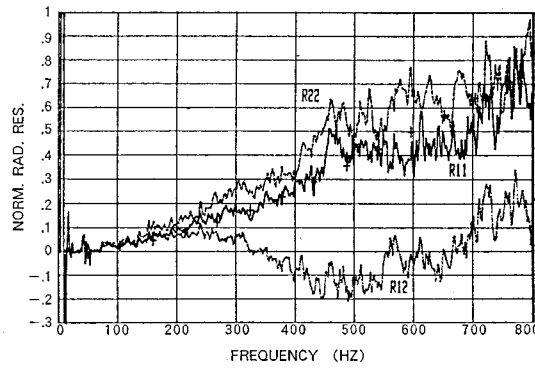


Fig. 10 Normalized self (R_{11} and R_{22}) and mutual (R_{12}) radiation resistances when the two loudspeakers are placed at location 3 and separated by 150 mm (normalization factor is $\rho c \pi a^2$).

it is essential to keep the primary and additional sources as close as possible.

6. CONCLUSION

Firstly, a relation between total radiated power and self and mutual radiation impedances of two sources was described. Using this relationship, an experimental method of determining self and mutual radiation resistances from measurements of radiated power and velocities of the sources was proposed.

As an example, radiation resistances of two loudspeakers located in a large meeting room were measured at frequencies below 800 Hz. The measured self radiation resistances were compared to that of a rigid piston in an infinite baffle. The measured results show similar trend to that of the rigid piston, but the former is larger than the latter at most frequencies. When the speakers are placed at a corner of the room, the self radiation resistances are larger and more irregular than when the speakers are placed far from side walls. When the distance between the speakers is increased, the mutual radiation resistance decreases and the zero-cross frequency is also shifted lower.

Since it is very difficult to numerically calculate the radiation impedances of a loudspeaker in an actual room, the method proposed here is very useful in evaluating the performance of loudspeakers and the effect of the room. Measured resistances of sources can also be used for an active noise control.

As a future study, a method of determining radia-

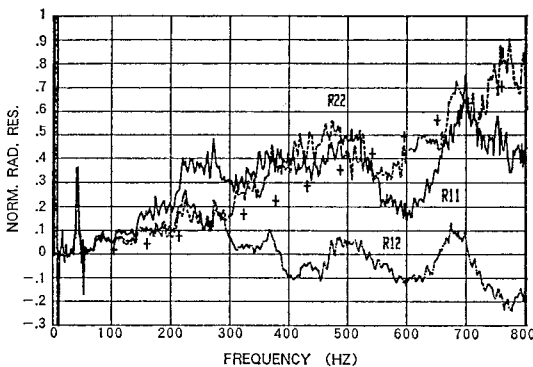


Fig. 9 Normalized self (R_{11} and R_{22}) and mutual (R_{12}) radiation resistances when the two loudspeakers are placed at location 2 (normalization factor is $\rho c \pi a^2$).

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tion resistances of loudspeakers from the measurement of input electrical power and diaphragm velocities will be investigated.

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