

Mutual Radiation Impedance of a Double-Disk Source and Its Effect on the Radiated Power*

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The acoustical interaction between two disks in an infinite baffle is discussed in terms of the mutual radiation impedance. The mutual radiation impedance is calculated for several different source geometries by numerically evaluating the Rayleigh integral. Then its effect on the radiated power of a two-way system with various Butterworth crossover networks is evaluated. When the two disks vibrate in phase, the increase in efficiency at the crossover frequency is less than 3 dB (which is achieved only in an ideal case) because of the increased size of the source. When the two disks vibrate $\pm 90^\circ$ out of phase, the total radiated power is equal to the power radiated from the two disks without the interaction. In this case, however, an excessive power is radiated from one disk and the same amount of power is absorbed by the other disk.

0 INTRODUCTION

In a multiway loudspeaker system two (or more) drivers radiate sound simultaneously near the system's crossover frequencies. Since the pressure generated by one driver also works as an acoustical load on the other driver, the radiation impedance of each driver is different from the one when it radiates sound by itself. This phenomenon has significant importance for the design of a crossover network, because the acoustical coupling between the drivers may affect the radiation efficiency of the system. This subject was recently discussed in Vanderkooy and Lipshitz [1] using two kinds of simplified models. They showed that the acoustical power of a two-point-source model is frequency dependent even though the input (electric) power into the system is frequency independent. Using the same model, they showed that the acoustical power output is simply the sum of the powers radiated by individual sources when the phase difference between them is $\pm 90^\circ$. In their second model, a frequency-independent polar pattern of $\cos \alpha$ was used as a crude approximation for a circular disk (α is the angle between the main axis and an observation point far from the source). In the present paper, however, a more realistic model (that is, without

the approximation of the directional characteristic $\cos \alpha$) comprising two rigid circular disks in an infinite baffle will be used.

The radiation problem of a planar source in an infinite baffle is solved by evaluating the well-known Rayleigh integral. Lord Rayleigh [2] calculated the self-radiation impedance of a single rigid disk in an infinite baffle by analytically integrating the pressure field in the immediate vicinity of the disk. Klopman [3] treats the case of two disks of equal size as their separation is varied. Lyon [4] discusses the mutual impedance of a vented loudspeaker from an analogous circuit point of view. The mutual radiation impedance of two rigid circular disks was obtained by Pritchard [5] by integrating the far-field directional characteristics with respect to the two angles of the spherical coordinate system. Arase [6] computed the mutual radiation impedance of equal shape and rectangular radiators by direct numerical integration. This direct numerical evaluation of the Rayleigh integral will be employed for the present problem. The application of the fast Fourier transform (FFT) method to radiation problems was recently discussed in Stepanishen and Benjamin [7] and Williams and Maynard [8]. This technique is powerful for evaluating the sound field over a fairly large area above the source. It was found, however, that the direct numerical evaluation technique is more appropriate for the present problem since it gives more accurate results than the FFT technique within a limited time.

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First, the effects of the radius ratio and the distance between the two disks on the mutual radiation impedance characteristics will be discussed. Then, using these mutual radiation impedances, the power responses of an ideal two-way loudspeaker system with a first-, second-, or third-order crossover network will be discussed.

1 MATHEMATICAL DISCUSSION

1.1 Geometry of the Model

The geometry of the model is shown in Fig. 1. Two rigid circular disks I and II are located in an infinite baffle with a center-to-center distance D . Disks I and II have radii R_1 and R_2 , surface areas S_1 and S_2 , and velocities v_1 and v_2 , respectively. The Cartesian coordinate system (x, y, z) with the x and y coordinates on the baffle will be used.

1.2 Mutual Radiation Impedance and Radiated Power

The forces f_1 and f_2 acting on disks I and II, respectively, are given by [9]

$$f_1 = z_{11}v_1 + z_{12}v_2 \quad (1)$$

and

$$f_2 = z_{21}v_1 + z_{22}v_2 \quad (2)$$

where z_{11} and z_{22} are the self-radiation impedances and z_{12} ($= z_{21}$ because of the reciprocity principle) is the mutual radiation impedance. The total radiation impedances z_1 and z_2 of the disks I and II are given by

$$z_1 = f_1/v_1 = z_{11} + z_{12}(v_2/v_1) \quad (3)$$

and

$$z_2 = f_2/v_2 = z_{22} + z_{12}(v_1/v_2) \quad (4)$$

The powers radiated by disks I and II are given as

$$1/2 f_1 v_1^* = 1/2 z_{11} |v_1|^2 + 1/2 z_{12} v_1^* v_2 \quad (5)$$

and

$$1/2 f_2 v_2^* = 1/2 z_{22} |v_2|^2 + 1/2 z_{21} v_1 v_2^* \quad (6)$$

where the asterisk denotes a complex conjugate. The total complex power is then given by

$$W = 1/2 z_{11} |v_1|^2 + 1/2 z_{22} |v_2|^2 + z_{12} |v_1 v_2| \cos \theta \quad (7)$$

where θ is the phase difference between v_2 and v_1 , $\arg(v_2/v_1)$. The effective powers are given by taking real parts of Eqs. (5)–(7). The last term of Eq. (7) is the power due to the acoustical coupling between the disks. If $\theta = \pm \pi/2$, this term becomes zero and there

is no coupling in terms of the total power. It should be noted, however, that there is an acoustical coupling as long as $|v_1 v_2| \neq 0$, even if $\theta = \pm \pi/2$, since the second term of Eq. (5) or (6) never becomes zero unless $z_{12} = 0$. This means that one disk must radiate more energy than the other to maintain the same total output.

By letting $v_2 = 0$ in Eq. (2), the mutual radiation impedance is given by

$$z_{12} = z_{21} = f_2/v_1 \quad (8)$$

This formula will be used to calculate the mutual radiation impedance between the two disks.

1.3 Numerical Evaluation of the Rayleigh Integral

When disk I vibrates with a velocity of v_1 (uniform on the surface S_1), the pressure at a point $(x, y, 0)$ on the surface S_2 is given by the Rayleigh integral

$$p(x, y, 0) = \frac{i\omega\rho}{2\pi} \int_{S_1} \frac{v_1 e^{-ikR}}{R} dS \quad (9)$$

where $R = [(x - x')^2 + (y - y')^2]^{1/2}$ is the distance between the surface element dS at a point $(x', y', 0)$ on the surface S_1 and the observation point $(x, y, 0)$ on the surface S_2 , k is the wavenumber, ρ is the density of air, and the time-dependent term $e^{i\omega t}$ is omitted for simplicity. If disks I and II are divided into small elements ΔS_m and ΔS_n of numbers M and N , respectively, Eq. (9) can be rewritten as an approximation,

$$p(x_n, y_n, 0) = \frac{i\omega\rho}{2\pi} \sum_{m=1}^M Q_m \frac{e^{-ikR_{mn}}}{R_{mn}} \quad (10)$$

where $R_{mn} = [(x_n - x'_m)^2 + (y_n - y'_m)^2]^{1/2}$, with points $(x'_m, y'_m, 0)$ and $(x_n, y_n, 0)$ at the centers of the elements ΔS_m and ΔS_n , respectively, and $Q_m = v_1 \Delta S_m$. Eq. (10) shows that disk I is comprised of a total of M point sources, each of which has a source strength of Q_m and is located at the point $(x'_m, y'_m, 0)$. Then the total force acting on disk II is given by

$$f_2 = \sum_{n=1}^N p(x_n, y_n, 0) \Delta S_n \quad (11)$$

The mutual radiation impedance between disks I and II is obtained by dividing f_2 by v_1 .

In the evaluation of the mutual radiation impedance, the surface of disk I was divided into small elements by J circles which have the radii of (jR_1/J) , $j = 1, 2, \dots, J$, and L radial lines which have angles of $(l \cdot 2\pi/L)$, $l = 1, 2, \dots, L$. The right-half surface of disk II was divided by M circles and N radial lines in a similar manner as for disk I (notations of M and N are different from the previous usage). The numbers used for J , L , M , and N were 10, 36, 10, and 18, respectively. In the frequency range of interest, the values of the

mutual radiation impedance did not show significant differences, even when the numbers for J , L , M , and N were increased to 14, 48, 14, and 27, respectively, ensuring the accuracy of the results.

2 RESULTS AND DISCUSSION

2.1 Mutual Radiation Impedance

Since our main interest is the coupling between the drivers of a loudspeaker system, the geometry of the source and the frequency range of interest will be limited more or less to practical values. Table 1 shows six cases of system geometries used for the calculation of the mutual radiation impedance. Case I is the standard with $D = 0.3$ m, $R_1 = 0.15$ m, and $R_2 = 0.05$ m. In cases II and III only the radius of disk II is changed, with $R_2 = 0.03$ m and 0.1 m, respectively. In cases IV and V only the distance between the two disks is changed from case I with $D = 0.2$ m and 0.4 m, respectively. Case VI is for $R_1 = R_2 = 0.15$ m with $D = 0.4$ m.

The normalized mutual radiation impedances are shown in Figs. 2–5. Figs. 2 and 3 are the resistances (real part) and reactances (imaginary part) for cases I–III; Figs. 4 and 5 are those for cases IV–VI. The normalization impedance is $\rho c \pi R_2^2$, where c is the sound velocity. This is equal to the radiation resistance of disk II at the high-frequency limit. The frequency axis is also normalized to $c/2\pi R_1$. In this case, the normalized frequency $kR_1 = 1$ corresponds to 364 Hz with $c = 343$ m/s and $R_1 = 0.15$ m. (For a woofer with a radius of 0.15 m, a crossover frequency around 300–400 Hz seems practical.)

When the mutual radiation impedance is normalized to $\rho c \pi R_2^2$, the evaluation of the coupling effect on the

total radiation impedance z_1 of disk I is easy. Eq. (3) is rewritten as

$$\begin{aligned} z_1 &= z_{11} + z_{12} \frac{v_2}{v_1} \\ &= z_{11} + z_{12} \left(\frac{R_1}{R_2}\right)^2 \\ &= \rho c \pi R_1^2 \left(\frac{z_{11}}{\rho c \pi R_1^2} + \frac{z_{12}}{\rho c \pi R_2^2} \right) \end{aligned} \tag{12}$$

where it is assumed that $R_1^2 v_1 = R_2^2 v_2$ (equal volume velocity). Since the normalized self-radiation impedance $z_{11}/\rho c \pi R_1^2$ is very often found in acoustics texts [10, 11], it can be directly compared to the values shown in Figs. 2–5 to evaluate the total radiation impedance and, hence, the radiated power from disk I. The other reason for normalizing the mutual radiation impedance to $\rho c \pi R_2^2$ is that it becomes less independent on the radius of disk II for small R_2/R_1 ratios or in the range $kR_1 \leq 1.0$, as seen in Figs. 2–5. The reason is that disk II may be considered a point source for $R_2/R_1 \leq 1/3$. For the standard case (case I), $z_{11}/\rho c \pi R_1^2 = 0.423 + 0.647i$ and $z_{12}/\rho c \pi R_2^2 = 0.212 - 0.074i$ at $kR_1 = 1.0$. This indicates that the radiation efficiency of disk I is increased by 50% and the acoustical mass

Table 1. Geometries used for the calculation of mutual radiation impedance.

Case	D (meters)	R ₁ (meters)	R ₂ (meters)
I	0.3	0.15	0.05
II	0.3	0.15	0.03
III	0.3	0.15	0.10
IV	0.2	0.15	0.05
V	0.4	0.15	0.05
VI	0.4	0.15	0.15

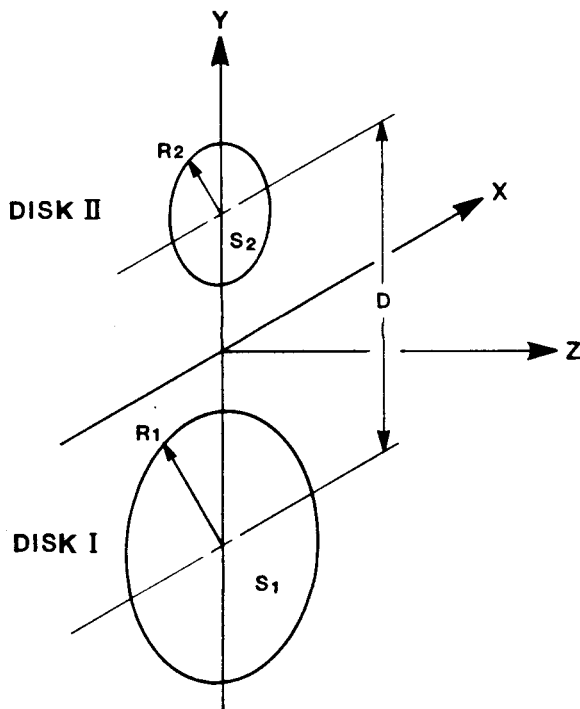


Fig. 1. Geometry of the model.

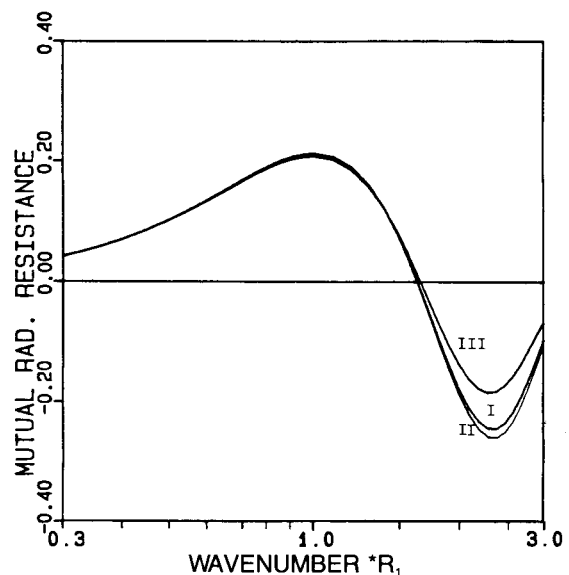


Fig. 2. Mutual radiation resistances of cases I–III normalized to $\rho c \pi R_2^2$.

load is reduced by 11% if the two disks vibrate in phase with the same volume velocity at this frequency. The mutual radiation resistance and reactance have the same order of magnitude and seem to satisfy the Kramers-Kronig relationship [12]. The peak frequencies of one characteristic correspond to the frequencies at which the second derivative of the other characteristic is zero. In the following sections, the effect of the mutual radiation impedance on the radiated power responses from double-disk sources will be discussed using the results given in Figs. 2-5.

2.2 Power Response of a Parallel-Driven Double-Disk Source

The simplest case that illustrates the effect of the mutual radiation impedance on the radiated power is the sound radiation from a parallel-driven double-disk source. In some loudspeaker systems two identical drivers are used in the same reproduction range to increase the efficiency or the power-handling capacity. In this case the on-axis responses are the same for the single-disk and the double-disk sources (except for the 3-dB difference). However, the radiated power responses, or the directional characteristics, are different because of the difference of the source geometry.

The geometry of case VI will be used to see how the radiated power response changes from the single-disk source to the double-disk source. The three normalized power responses will be calculated to see the effect of the mutual radiation impedance. Those are

$$PR1 = 1/2 \text{Re}(z_{11}|v_1|^2 + z_{22}|v_2|^2)/w_0 \quad (13)$$

$$PR2 = 1/2 \text{Re}(z_{11}|v_1|^2 + z_{22}|v_2|^2 + 2z_{12}|v_1v_2|)/w_0 \quad (14)$$

$$PR3 = 1/2 \text{Re}(z_{11}|v_1|^2 + z_{22}|v_2|^2 - 2z_{12}|v_1v_2|)/w_0 \quad (15)$$

where w_0 is the power radiated from disks I and II at very low frequencies ($kR_1 = kR_2 \leq 0.1$), neglecting the mutual radiation impedance. The velocities v_1 and v_2 are the same, and they are inversely proportional to

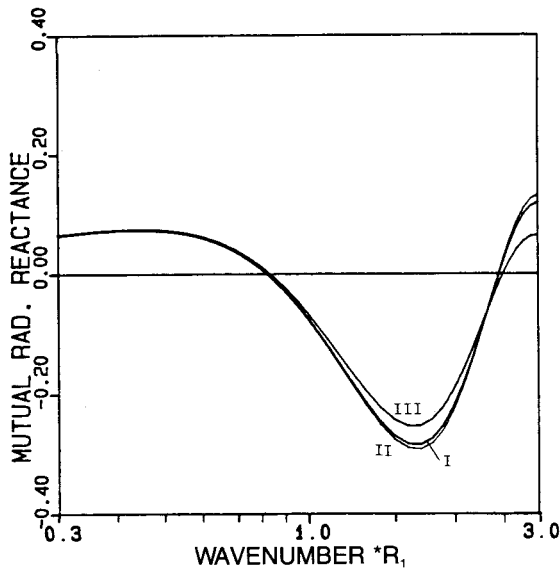


Fig. 3. Mutual radiation reactances of cases I-III normalized to $\rho c \pi R_2^2$.

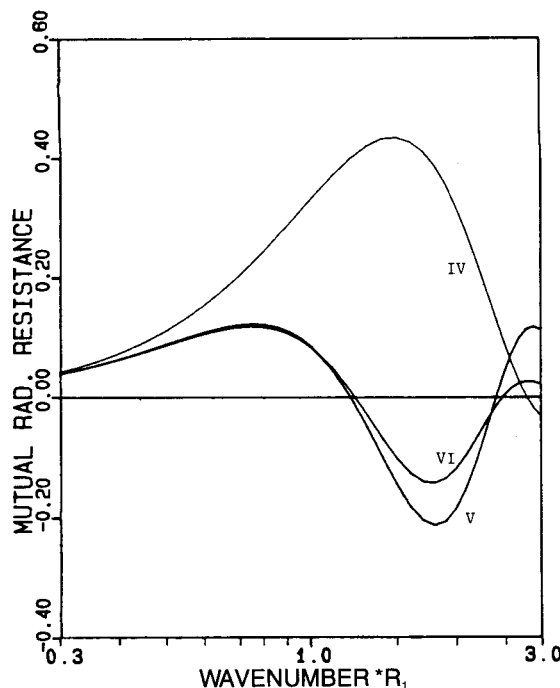


Fig. 4. Mutual radiation resistances of cases IV-VI normalized to $\rho c \pi R_2^2$.

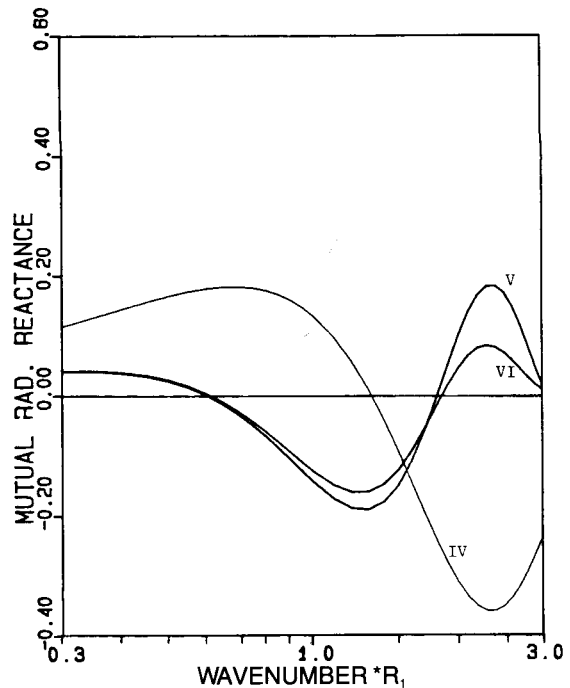


Fig. 5. Mutual radiation reactances of cases IV-VI normalized to $\rho c \pi R_2^3$.

the frequency, which is a reasonable assumption for a moving voice-coil direct-radiator loudspeaker. The response PR1 is the power response of the double-disk source when the mutual radiation impedance is neglected. The responses PR2 and PR3 are the total power responses when the two disks vibrate in phase and out of phase to each other. Fig. 6 shows the results.

The response PR1 approaches 0 dB and the response PR2 approaches 3 dB as the frequency decreases. The 3-dB increase of the radiated power at low frequencies is due to the mutual radiation impedance. Actually, z_{11} is equal to z_{12} in the low-frequency range since the force acting on disk I due to the vibration of disk II is equal to the force due to its own vibration when the wavelength is much larger than the system geometry. The decrease of PR1 for increased frequencies is due to the fact that the radiation from each disk is gradually becoming directional or beamy. The response PR2 indicates that this effect is more dominant when the interaction between the two disks is taken into account. The response PR2 indicates that a double-woofer source with an effective diameter of 0.3 m each will decrease its power response by 3.0 dB at 364 Hz ($kR_1 = 1.0$) when two woofers are 0.4 m apart in an infinite baffle. It is the author's experience that when two woofers are used in parallel, the system sounds boomier in some cases, even though the on-axis response seems unchanged from that of a single-woofer system. The response PR3 is impractical since the on-axis pressure response is null over the entire frequency range. It should be noted, however, that the power response is very small only at very low frequencies. Actually, PR3 is higher than PR2 in the range where the mutual radiation resistance becomes negative.

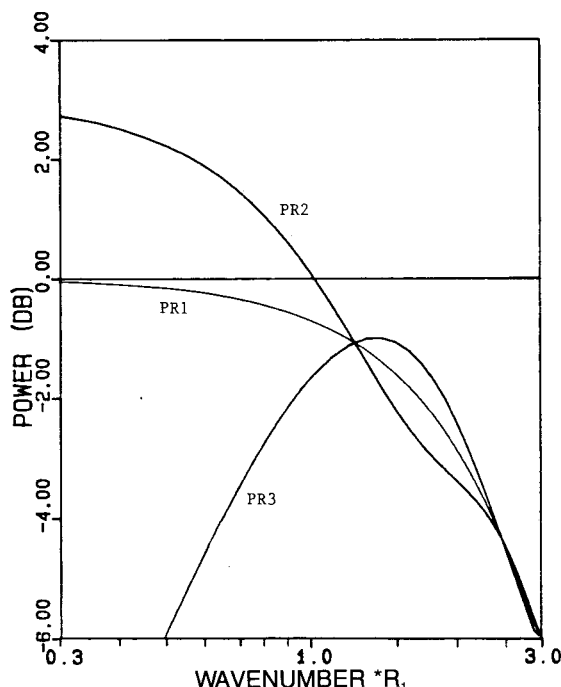


Fig. 6. Power responses of double-disk source with geometry of case VI.

2.3 Power Responses of a Two-Way System with a Second-Order Butterworth Crossover Network

As a second example to see the effect of the mutual radiation impedance on the radiated power, a two-way system with a Butterworth crossover network will be considered. The Butterworth crossover networks are convenient for a discussion of the power response since the electric input power to the system is constant.

The low-pass and high-pass amplitude responses of the N th-order Butterworth filters are given by

$$|G_L(\omega)| = \frac{1}{[1 + (\omega/\omega_0)^{2N}]^{1/2}} \quad (16)$$

and

$$|G_H(\omega)| = \frac{(\omega/\omega_0)^N}{[1 + (\omega/\omega_0)^{2N}]^{1/2}} \quad (17)$$

where ω_0 is the crossover frequency. The velocity amplitudes of disks I and II can be represented by

$$|v_1| = \frac{|G_L(\omega)|}{\omega} \quad (18)$$

and

$$|v_2| = \left(\frac{S_1}{S_2}\right) \frac{|G_H(\omega)|}{\omega} \quad (19)$$

The coefficient S_2/S_1 is necessary to compensate for the difference of the area between two disks, and the factor $1/\omega$ is used for the same reason as mentioned in the previous section. The phase difference between v_1 and v_2 is equal to the phase difference of $G_L(\omega)$ and $G_H(\omega)$, which is given as

$$\arg\left(\frac{v_2}{v_1}\right) = \arg\left(\frac{G_H}{G_L}\right) = N\left(\frac{\pi}{2}\right) \quad (20)$$

This is independent of frequency. For even orders of N , the velocities are in phase ($N = 4, 8, \dots$) or 180° out of phase ($N = 2, 6, \dots$). For odd orders of N , the phase difference is equal to $+\pi/2$ or $-\pi/2$. At first the effect of the mutual radiation impedance will be discussed using the second-order Butterworth crossover network since, as Eq. (7) indicates, the radiated power due to the mutual radiation impedance is zero for an odd order of N .

The geometries of the system used for calculation are cases I, II, and V shown in Table 1. The normalized crossover frequency $k_c R_1$ is equal to 1 unless otherwise stated. The same normalized power responses [Eqs. (13)–(15)] will be calculated to see the effect of the mutual impedance.

In the present case the normalizing power w_0 is the power radiated from disk I only at very low frequencies.

The response PR2 is the total power response when one of the polarities is reversed ($\cos \theta = 1$). The response PR3 is the total radiated power for normal polarity connection ($\cos \theta = -1$).

Fig. 7 shows the power responses of a two-way system with the geometry of case I (standard case) and a crossover frequency at $k_c R_1 = 1.0$. The response PR1 shows that the radiated power is not constant, even when the effect of the mutual radiation impedance is neglected. The reason is again the beaming effect of radiation from a disk at mid- to high frequencies. Actually, this is one of the reasons why a multiway loudspeaker system must be used for a high-fidelity loudspeaker system. The power response, when the two disks vibrate in phase (PR2), has a peak level of approximately 1.7 dB at around $kR_1 = 0.75$ (270 Hz), which is much lower than the 3-dB hump in the pressure response at the crossover frequency for the in-phase connection. The maximum increase of radiated power due to the interaction is only approximately 2 dB. As the response PR3 shows, the system has a trough of only 4 dB, even though the two disks vibrate 180° out of phase. It should be noted that much more power is radiated in this case than imagined from the zero on-axis pressure response at the crossover frequency.

The result for a smaller second disk (case II) with the crossover frequency at $k_c R_1 = 1.0$ is shown in Fig. 8. The response PR1 is closer to the 0-dB line than for case I. Since disk II is smaller, the beaming effect for the same frequency range is smaller for this case. The effect due to the mutual radiation impedance on the radiated power is almost identical with that for case I, as expected from the characteristics of the mutual radiation resistances for the two cases. The result for case V is shown in Fig. 9. When the distance between two disks is increased from 0.3 to 0.4 m, the mutual radiation impedance becomes smaller and consequently

its effect on radiated power also becomes small. In this case the responses PR2 and PR3 are closer to PR1 than for case I.

The results for case I with the crossover frequencies at $k_c R_1 = 0.8$ and 1.5 are shown in Figs. 10 and 11, respectively. When the crossover frequency is lowered, the effect of the mutual radiation impedance on the radiated power responses increases. The opposite occurs for the higher crossover frequencies. Lowering the crossover frequency is analogous to shortening the distance between two disks with respect to the effect of the mutual radiation impedance.

2.4 Power Responses of a Two-Way System with First- and Third-Order Butterworth Crossover Networks

For odd-order Butterworth crossover networks, the additional radiated power due to the mutual radiation impedance is zero, as Eq. (7) indicates. The power responses of individual disks, however, are significantly affected by the mutual radiation impedance. The following two power responses, as well as PR1 of Eq. (13), will be evaluated in this section:

$$PR4 = 1/2 \text{Re}(z_1 |v_1|^2 + z_{12} v_1^* v_2) / w_0 \quad (21)$$

$$PR5 = 1/2 \text{Re}(z_2 |v_2|^2 + z_{12} v_1^* v_2) / w_0 \quad (22)$$

The responses PR4 and PR5 are the normalized radiated power characteristics of disks I and II, respectively. The relationship $PR1 = PR4 + PR5$ holds for the entire frequency range.

The power responses of a two-way system with the geometry of case I (standard case) and the crossover frequency of $k_c R_1 = 1.0$ is shown in Fig. 12. The polarity connection is normal and, therefore, disk II

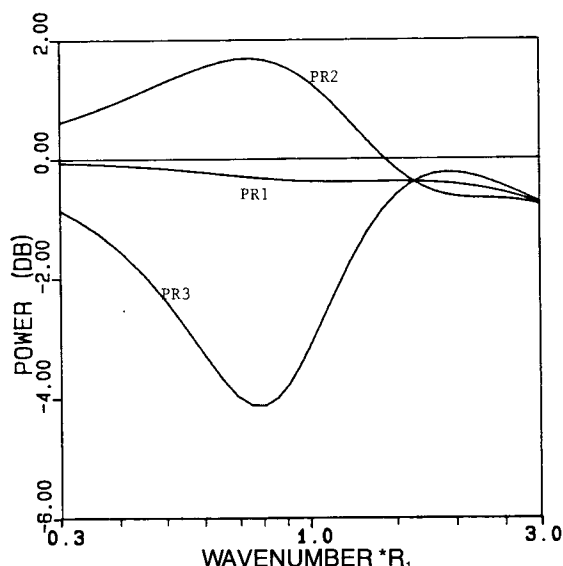


Fig. 7. Power responses of two-way system with geometry of case I and second-order Butterworth crossover network ($k_c R_1 = 1.0$).

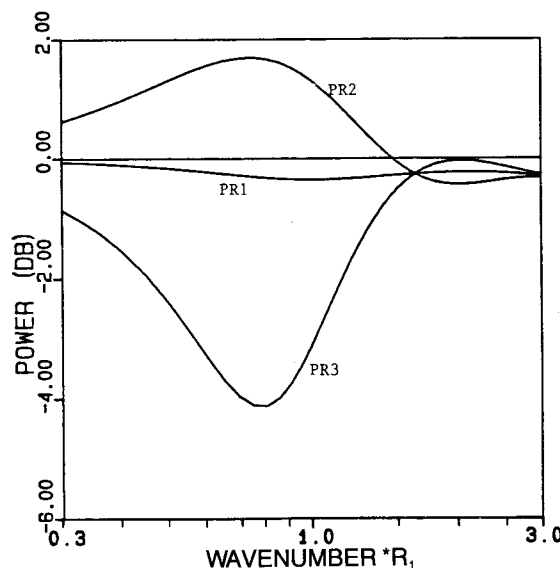


Fig. 8. Power responses of two-way system with geometry of case II and second-order Butterworth crossover network ($k_c R_1 = 1.0$).

leads $\pi/2$ in phase. The response PR1 is slightly different from the result shown in Fig. 7 because of the difference of the filter slopes. It is surprising that the radiated power levels of disks I and II are approximately the same at low frequencies (around $kR_1 = 0.3$). The reason is that a disk which is advanced in phase radiates extra energy and the other disk radiates less by the same amount, keeping the total power output equal to the power for the case when the interaction is neglected. The excess power radiated from disks I and II is $-x_{12} |v_1 v_2|$ and $+x_{12} |v_1 v_2|$, where x_{12} is the imaginary part of the mutual radiation impedance. Since x_{12} is positive

in the low-frequency region, disk II radiates more energy than it is supposed to do for the isolated case. When one of the polarity connections is changed so that disk I leads in phase, the situation is completely reversed. The result is shown in Fig. 13. Now, at low frequencies, disk I radiates more energy than it does when it is alone, and disk II absorbs energy rather than radiates it.

The forces acting on disks I and II are also drastically changed by reversing the polarity. Mostly, the mechanical impedance of a transducer is much larger than the acoustical impedances. In some cases, however,

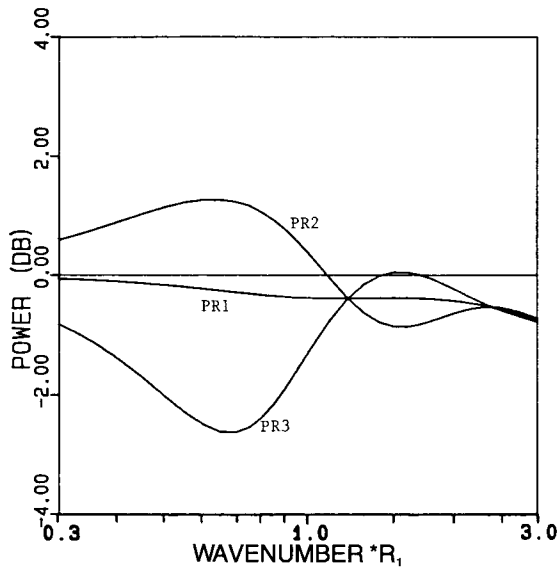


Fig. 9. Power responses of two-way system with geometry of case V and second-order Butterworth crossover network ($k_c R_1 = 1.0$).

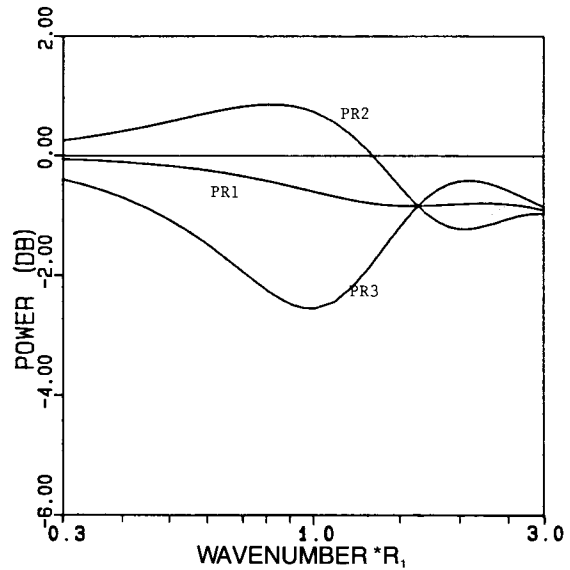


Fig. 11. Power responses of two-way system with geometry of case I and second-order Butterworth crossover network ($k_c R_1 = 1.5$).

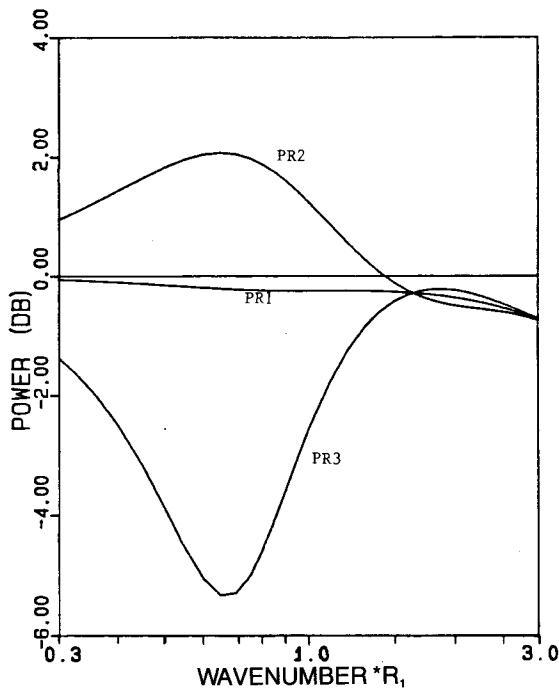


Fig. 10. Power responses of two-way system with geometry of case I and second-order Butterworth crossover network ($k_c R_1 = 0.8$).

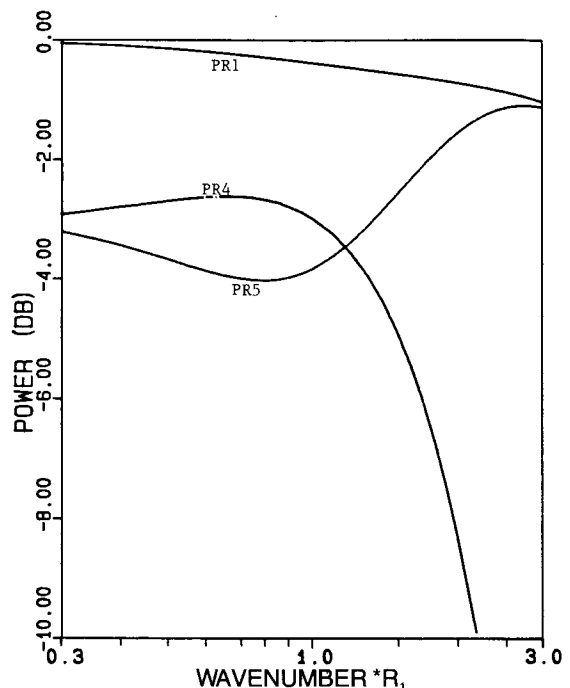


Fig. 12. Power responses of two-way system with geometry of case I and first-order Butterworth crossover network ($k_c R_1 = 1.0$).

the effect of the acoustical load on the vibration of the transducer diaphragm may not be negligible and, if this is the case, the polarity relationship of the transducers ($\sin \theta = +1$ or -1) may have some effects other than just changing the directivity patterns.

Figs. 14 and 15 show the results when a third-order Butterworth crossover network is used with the normal polarity connection ($\sin \theta = -1$) and the reversed po-

larity connection ($\sin \theta = +1$), respectively. Since v_1 and v_2 decay quickly outside their passbands, the effect of the mutual radiation impedance is smaller than in the previous case. For the normal polarity relationship, disk II absorbs energy in the low-frequency region. The frequency at which the two disks radiate the same amount of energy is different for the normal polarity case compared to the reversed polarity case.

3 CONCLUSION

The acoustical coupling between two disks in an infinite baffle was discussed in terms of mutual radiation impedance. The mutual radiation impedance was calculated for several cases of source geometries by numerically evaluating the Rayleigh integral. The mutual radiation resistance and reactance take negative values as well as positive values. Meanwhile, the self-radiation resistance and low reactance of a flat disk is always positive. In the very low-frequency region the mutual radiation impedance is equal to the self-radiation impedance of one disk when they are properly normalized. Consequently the radiation efficiency increases by 3 dB when the two disks vibrate in phase with the same volume velocity. This is not true for the frequency range where the size of the source and the wavelength become comparable.

Power responses of a two-way system with Butterworth crossover networks were discussed taking into account the acoustical coupling between the two disks. The on-axis pressure response has a 3-dB hump at the crossover frequency when the polarity is adjusted so that the two disks vibrate in phase with each other. However, the total radiated power has a smaller hump

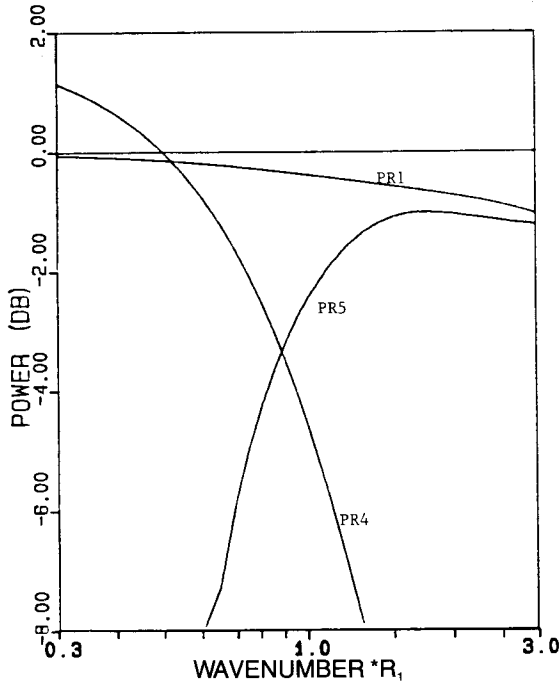


Fig. 13. Power responses of two-way system with geometry of case I and first-order Butterworth crossover network ($k_c R_1 = 1.0$) when tweeter (or woofer) polarity is reversed.

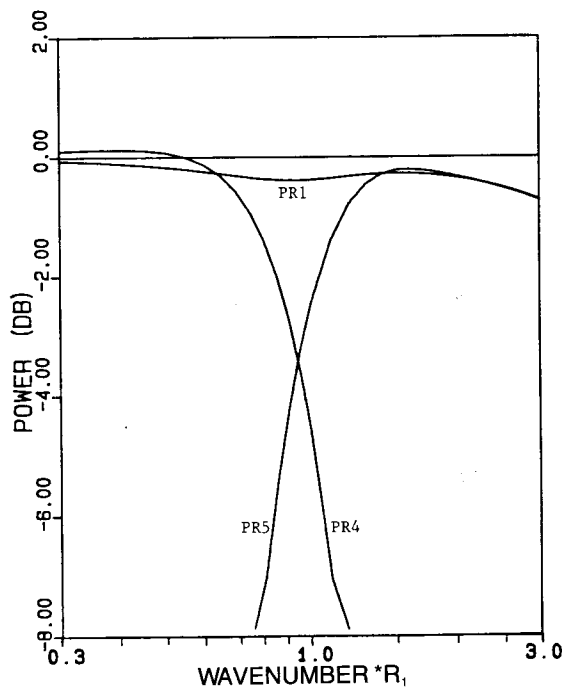


Fig. 14. Power responses of two-way system with geometry of case I and third-order Butterworth crossover network ($k_c R_1 = 1.0$).

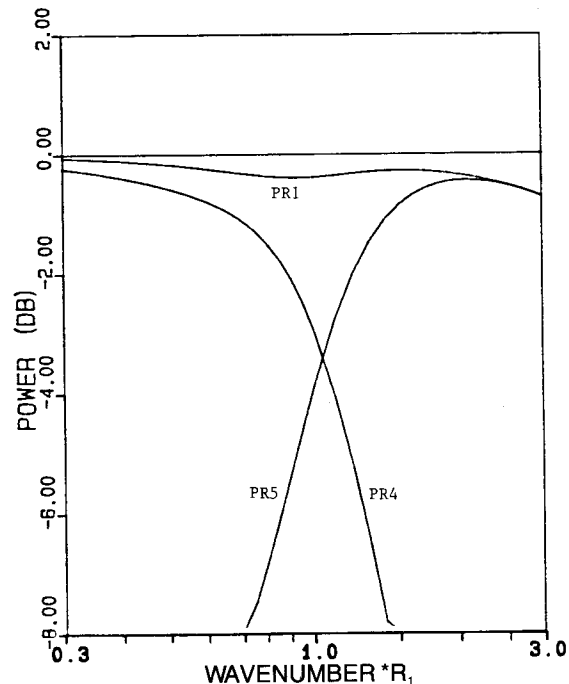


Fig. 15. Power responses of two-way system with geometry of case I and third-order Butterworth crossover network ($k_c R_1 = 1.0$) when tweeter (or woofer) polarity is reversed.

at a lower frequency for a normal configuration of the source. The power response of the system is dependent on the individual disk radius, the distance between them, and the crossover frequency, which must be calculated numerically for each case. When the two disks vibrate $+90^\circ$ or -90° out of phase with each other, there is no effect of the mutual radiation impedance on the total radiated power. However, the radiated power (in other words, the radiation impedance) of each disk is significantly changed. The real and imaginary parts of the mutual radiation resistance are added as the imaginary and real parts, respectively, to the self-radiation impedance. Other than changing the directional characteristics of the system, the effect of this type of interaction on the sound radiation is not known. It would be interesting to determine whether the intermodulation distortion is affected by the phase relationship between the transducers ($\theta = +\pi/2$ or $-\pi/2$).

It was demonstrated that the power responses of a rather ideal multiway loudspeaker system can be evaluated by the present method. In the crossover network design, if any attention is paid to the power response of the system, the effect of the mutual radiation impedance should be carefully evaluated.

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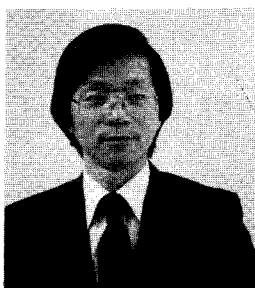
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